1. Problem 1

(This problem is taken from the 1997 final exam.) In January 1998 the S&P 500 index was 950, and the January 1999 S&P 500 futures price was 983.25. In January 1998 a 30-year Treasury bond sold for 130, and the January 1999 futures price was 122.85. The one-year riskless interest rate was 5%.

(a) Use the formula relating spot and futures prices to show how it is possible that the S&P 500 futures price is higher than the spot price, while the Treasury bond futures price is lower than the spot price.

Answer: \( F_0 = (1 + R_f - d)S_0 \), where \( d \) is the dividend/price or coupon/price ratio. This implies that \( F_0 - S_0 = (R_f - d)S_0 \).

The forward price is higher than the spot price when \( R_f > d \). The forward price is lower than the spot price when \( R_f < d \).

(b) Do these numbers tell you anything about:

i. the presence of normal backwardation or contango in the stock and bond futures markets;

Answer: These numbers in the problem do not provide useful information for identifying the presence of normal backwardation or contango in the stock and bond futures markets. To answer this question, we would need to know the expected return on stocks and the expected return on bonds, or equivalently, the expectation of the future spot prices. Though not specifically stated here, we know from historical data that stocks and long-term bonds have higher expected returns than short-term bonds. Therefore, we would expect normal backwardation in these markets, i.e. \( E_0 \left( \frac{S_1 + D}{S_0} \right) > 1 + R_f \).

ii. the dividend-price and coupon-price ratios in the stock and bond markets;

Answer: \( F_0 = (1 + R_f - d)S_0 \) implies that \( d = 1 + R_f - \frac{F_0}{S_0} \).

The dividend/price ratio is \( 1 + 0.05 - \frac{983.25}{950} = 0.015 \) or 1.5%.

The coupon/price ratio is \( 1 + 0.05 - \frac{122.85}{130} = 0.105 \) or 10.5%.

iii. expected returns on stocks and bonds?

If they do, explain how and give an explicit numerical calculation. If they do not, explain why not.

Answer: The data does not provide information about expected returns on stocks and bonds since there is no information about the expected future spot rates.

2. Problem 2

(This problem is taken from the 2000 final exam.) At the beginning of January the yields on US Treasury securities trading are as follows: Six-month Treasury bills 6%, one-year Treasury bills 5.75%, 2-year notes 5.5%, 3-year notes 5.25%, 4-year notes 5%. You may assume that each of these securities is trading at par. You work for Massachusetts Middleman Bank and are arranging a standard 4-year
interest-rate swap with notional amount $200 million. At the end of each year for 4 years, the floating payer (the bank’s customer) will pay the notional amount times the 1-year Treasury bill rate at the start of the year. In exchange, at the end of each year for 4 years, the fixed payer (the bank) will pay the notional amount times the fixed rate specified in the swap contract.

(a) Assume that neither the bank nor its customer has any risk of default. Assume that the swap is priced so that no money need change hands at the beginning of the swap. What must be the fixed rate in the swap contract?

**Answer:** In order for no money to change hands at the beginning of the swap, the present value of the future net payments made by the two parties must be equal to zero. As was shown in class (Lecture 19), this is assured by setting the fixed rate on the swap to the yield to maturity on a par coupon-bond with the same maturity. Thus, the fixed rate on the swap, \( r_{\text{fixed}} \), must be equal to \( Y_{4,t} = 5\% \).

(Note: In the solution above we assumed that the yields to maturity quoted in the problem correspond to coupon bonds which pay coupons at an annual frequency - i.e. the same frequency as the swap payments are made - which is not exactly correct since U.S. Treasury obligations make coupon payments on a semi-annual basis.)

(b) One year later, interest rates of all maturities are unchanged. What is the value of the swap contract to the bank?

**Answer:** The present value of the swap payments (i.e. the present value of the differences in variable and fixed payments) is given by:

\[
S_{t+1} = PV_{\text{variable}}(t+1) - PV_{\text{fixed}}(t+1) = N - \left\{ \frac{N \cdot r_{\text{fixed}}}{(1 + Y_{3,t+1})} + \frac{N \cdot r_{\text{fixed}}}{(1 + Y_{3,t+1})^2} + \frac{N \cdot (1 + r_{\text{fixed}})}{(1 + Y_{3,t+1})^3} \right\}
\]

From this formula we can see that the present value of the swap at \( t+1 \) would be equal to zero if an only if the fixed rate on the swap were equal to \( Y_{3,t+1} \), or equivalently, \( Y_{3,t} \) (the yield curve remains unchanged between \( t \) and \( t+1 \)). Since the fixed rate is different from \( Y_{3,t} \) there will be a net payment between the two parties.

The fixed payments are discounted at \( Y_{3,t+1} \) since this is the prevailing yield to maturity on a coupon bond with maturity equal to the maturity of the swap, which is now 3 years. Using the fact that the yield curve remains unchanged between \( t \) and \( t+1 \) and that \( r_{\text{fixed}} = Y_{4,t} \) we have:

\[
S_{t+1} = PV_{\text{variable}}(t+1) - PV_{\text{fixed}}(t+1) = N - N \left\{ \frac{Y_{4,t}}{(1 + Y_{3,t})} + \frac{Y_{4,t}}{(1 + Y_{3,t})^2} + \frac{1 + Y_{4,t}}{(1 + Y_{3,t})^3} \right\}
\]

\[
= \$200M \cdot \left[ 1 - \frac{0.05}{1.0525} - \frac{0.05}{1.0525^2} - \frac{1.05}{1.0525^3} \right] = \$1.355M
\]

So the computation shows that the present value of the swap from the perspective of the bank is \$1.355 million. If the customer wanted to cancel the swap agreement (“unwind the swap”), it would have to pay the bank \$1.355 million to do so. This makes sense since at \( t+1 \) the swap agreement is only a three-period agreement whereas the fixed payments which are the bank’s obligation remain fixed at 5%, which is below the three year discount rate (5.25%).
(c) Explain how this value can change over time even when interest rates do not change over time.

**Answer:** The value of the swap to the bank is given by the difference in the present values of the variable payments (received by the bank) and the fixed payments (paid out by the bank). In this case, the fixed payment rate is fixed at \( Y_{4,t} = 5\% \), the rate ensuring that no money changes hands at \( t \). If we were to enter into a similar agreement at time \( t + 1 \) - this time for three periods so that the maturity of the new and old swaps are identical - the fixed rate would be set at \( Y_{3,t+1} = Y_{3,t} = 5.25\% \) forcing the bank to pay higher fixed payments. Consequently, the bank has managed to lock in a favorable fixed payment rate, which is reflected by decline in the present value of the fixed payments between \( t \) and \( t + 1 \). Thus, we can see that the condition guaranteeing that the value of the swap remains 0 over the life of the swap is \( Y_{k-i,t+i} = Y_{k,t} \) for all \( i \in (0, k-1) \).

(d) In practice both banks and their customers can default. Does default risk have a larger effect on swap rates, or on the yields of long-term corporate bonds? Explain your answer.

**Answer:** Default risk has a larger effect on the yields of long-term bonds than on swap rates. Bonds require the payment of face value at maturity, whereas swaps do not require the exchange of the notional amount at maturity. This makes default risk less of a problem for swaps.

### 3. Problem 3

(a) Why is there no futures market in cement?

**Answer:** Cement prices tend to be quite stable, and therefore, quite predictable. As a result, there is little need to hedge against changes in cement’s price, since such changes, if any, will likely be minor. The implications of this are that a cement future market would see very little volume. In addition, the cement market is quite highly concentrated (an oligopoly), which means that individual firms have market power. As a result, the prices in the spot market may be manipulated by the suppliers, which would affect the values of futures contracts.

(b) What is the difference in cash flow between short-selling an asset and entering a short futures position?

**Answer:** There are two components to the cash flow - the immediate effect from entering into the transaction and the associated margin requirements. When an asset is sold short the agent immediately receives the value of the asset; conversely, when entering a short futures position no money changes hands at the onset.

Secondly, when one sells an asset short, one typically must put up the entire value of the asset (plus about 2%, or 102% of the value total) as collateral. This money is given to the intermediary lending the stock. When taking a short future position, however, one normally needs to put up only about 10% of the total value of the contract as margin (margins tend to range from 5% to 15%) to the clearinghouse. In both cases, however, the clearinghouse or lender credits or deducts money from the margin account or the collateral as soon as there is any change in the price of underlying asset, meaning the realization of gains and losses is immediate.

(c) Evaluate the criticism that futures markets siphon off capital from more productive uses.
Answer: Since entering into a futures transaction does not require immediate outlays of capital, with the exception of the small amount required to satisfy the variation margin (about 10% of the notional amount of the futures contract), futures markets do not siphon off capital from productive uses. The margin capital can also be posted in the form of liquid interest-bearing bonds.

Furthermore, futures markets are highly valuable in that they do not force producers to bear commodity price risk, and that they provide a simple means of reducing risk in general. In the case of commodities markets, it is likely that many producers would not be willing to take the risk of massive price decreases in the commodity they produce, and therefore that many might decide not to produce at all. In fact, by allowing such producers to “sell” their risk to investors in the form of futures contracts, total production is likely to be increased.

4. Problem 4

(a) Suppose the current valuation of each tranche is the monthly S&P 500 price index series in Assignment7_Data.xls. Consider values of $K$ from $K = 1$ to $K = 12$ (when $K = 1$ the entire portfolio is repriced every month, and when $K = 12$ each tranche gets repriced only once a year). Compute the series of reported portfolio values for each of the values of $K$.

Answer: See computations in Assignment7_Solution.xls posted on the course website.

(b) Compute the monthly price return series for each of the twelve scenarios, and report their annualized standard deviations. What happens as $K$ increases? Explain.

Answer: As $K$ increases the monthly standard deviation of stock returns declines. In general, smoothing prices also tends to smooth out the returns introducing a downward bias in the estimated monthly volatility. Simultaneously, increasing $K$ increases the degree of positive serial correlation in returns. Therefore, just as in Asness, Krail and Liew (JPM 2001), we can verify that the annualized quarterly volatility exceeds annualized monthly volatility in the presence of price smoothing.

(c) Compute and report the betas of the twelve return series with the S&P500 price return series. What happens as $K$ increases? Explain.

Answer: As $K$ increases the (contemporaneous) beta estimates decline. By smoothing the portfolio price series the price adjustments (i.e. price returns) in the S&P500 are only gradually incorporated in the price return of the portfolio, and the rate at which they are incorporated declines as $K$ increases. Since $\beta$ measures the contemporaneous reaction of the portfolio to returns on the S&P500 we would expect it to decline as the adjustment to contemporaneous shocks to price becomes slower, i.e. as $K$ increases.

(d) Suppose Harvard Management Company is concerned that stale pricing might be affecting its ability to judge the risks of investments in hedge funds and private equity funds. What can HMC do to improve its risk assessments?

Answer: In order to correctly account for risk of investments suspected of being affected by stale pricing (which induces spurious positive serial autocorrelation) one can include lagged index returns in market regressions. That is, instead of determining the $\beta$ by running a regression of the excess asset returns on the contemporaneous excess market return, one should include
lagged terms, as in:

\[ R_{i,t} - R_{f,t} = \alpha + \beta_{0,i,m} (R_{m,t} - R_{f,t}) + \beta_{1,i,m} (R_{m,t-1} - R_{f,t-1}) + \]
\[ + \beta_{2,i,m} (R_{m,t-2} - R_{f,t-2}) + \ldots + \varepsilon_t \]

By computing the sum of the contemporaneous and lagged betas, as in the Asness, Krail and Liew (JPM 2001) article, we can correctly account for the distortions in risk assessments due to sluggish adjustment of prices induced by illiquidity or “creative” marking-to-market.
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**Effects of stale pricing on sigma**

**Effects of stale pricing on beta**