Multiunit Auctions
Lecture notes for Market Design, Professor Susan Athey

Thanks to Larry Ausubel and Peter Cramton for sharing their notes.

Literature

• Main References for this Lecture
  – “An Efficient Ascending-Bid Auction for Multiple Objects,” by Lawrence Ausubel
  – “Demand Reduction and Inefficiency in Multi-Unit Auctions,” by Larry Ausubel and Peter Cramton
  – “The Ascending Auction Paradox” by Larry Ausubel and Jesse Schwartz
  – “Auctioning Many Divisible Goods” by Larry Ausubel and Peter Cramton

• Other work
  – Maskin and Riley (1989): Independent private values, multiple identical objects, optimal auctions
  – Dasgupta and Maskin (1999): Efficient sealed bid auction with interdependent valuations
  – Gul and Stacchetti, etc.: complete information, multiple objects, relate ascending auctions to Walrasian prices
Auctioning similar items

- Treasury bills
- Stock repurchases
- Telecommunications spectrum
- Electric power
- Emissions permits
- Petrochemical products

Ways to auction many similar items

- **Sequential**
  - Sequence of English auctions (auction house, cars)
  - Sequence of Dutch auctions (fish, flowers)

- **Sealed Bid, Simultaneous**
  - Uniform Price (Pay market-clearing price)
  - Discriminatory (Pay your bid)
  - Vickrey (Pay social opportunity cost)

- **Ascending-bid**
  - Simultaneous ascending-bid (FCC spectrum)
  - Ascending-clock auction with quantity choice
  - Alternative ascending-bid auction

- **Optimal auction**
  - Maskin & Riley 1989
Sealed-bid, Simultaneous

- Bidders submit demand schedules, intersect supply and demand, allocate to highest demand bids
- What varies: pricing rule
  - Pay-your-bid auction (traditional Treasury practice)
  - Uniform-price auction (Milton Friedman 1959)
  - Vickrey auction (William Vickrey 1961)

**Pay-Your-Bid Auction:**
All bids above $P_0$ win and pay bid
**Uniform-Price Auction:**
All bids above $P_0$ win and pay $P_0$

**Vickrey Auction:**
All bids above $P_0$ win and pay opportunity cost
Vickrey Auction: Example

- Efficient Allocation
  - Highest marginal values
  - Note that mgl values are declining
- Assignment of prices to specific units somewhat arbitrary; what matters is price for whole bundle
- Vickrey rules:
  - Remove winning bidder for bundle from the game
  - Figure out values who would have received these unit, yields price for bundle
- Social opportunity cost of items won
- Truth telling is equilibrium

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Vickrey Auction: Incentives

- Vickrey rules:
  - Remove winning bidder for bundle from the game
  - Figure out values who would have received these unit, yields price for bundle
- Like a second-price auction:
  - Each bidder internalizes social objective
  - All bids only affects whether you win particular items, do not affect how much you pay on any item
- What happens to incentives if I don’t remove all of bidder’s bids when setting prices?

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Payment rule affects behavior

![Diagram showing Price vs. Quantity with Pay-Your-Bid, Uniform-Price, Vickrey, Residual Supply, and Demand]

Comparing Auctions

- What are Objectives?
  - Efficiency
  - Revenue
  - Simplicity/transparency
- Efficiency
  - FCC: those with highest values win
- Revenue maximization
  - Treasury: sell debt at least cost
Main Results: Comparing Auctions

- Uniform-price auction inevitably leads to bid shading and inefficiency
- Revenue ranking of pay-your-bid auction and uniform-price auction is ambiguous
- Vickrey auction may outperform both
  - Efficiency (always)
  - Seller revenues (sometimes)
  - But complexity concerns, vulnerability to collusion
- New ascending-bid auctions may perform even better

“Demand Reduction and Inefficiency in Multi-Unit Auctions,” by Larry Ausubel and Peter Cramton

Uniform-price auction fallacy

“You need only know the maximum amount you are willing to pay for different quantities.”
- Milton Friedman, on strategy in the uniform price auction (Wall Street Journal 1991)

“All of that is eliminated if you use the [uniform-price] auction. You just bid what you think it’s worth.”
- Merton Miller, on the absence of bid shading in uniform price auction (New York Times 1991)
- Note: top 5 bidders buy 50% of shares
Inefficiency Theorem

_In any equilibrium of uniform-price auction, with positive probability objects are won by bidders other than those with highest values._

- Winning bidder influences price with positive probability
- Creates incentive to shade bid
  - Shift in stated demand curve at a particular point has two effects:
    - Marginal effect: win fewer units
    - Inframarginal effect: pay less on all units
- Incentive to shade increases with additional units
  - Consider the case where a particular part of the demand curve sets the price
  - The more units below that, the greater the inframarginal effect
- Differential shading implies inefficiency
  
  _Exceptions:_
  
  Pure common value
  Bidders demand only a single unit

Inefficiency theorem and bid shading

- Qualitative appearance of optimal bid strategy
Inefficiency from differential shading


“Irrespective of whether the [uniform-price] auction would prove superior to the current practice, the Agencies believe that there is an auction technique that may be superior to both types of sealed-bid auction techniques discussed above. This is an ascending-price, open auction system, which will be feasible for the first time once the auctions are automated. Auction theory suggests that, in general, Treasury revenue would not suffer, and indeed might increase, in the switch to an open, ascending-price system.” pp. 14-15

“Bidders would state their demands as the auctioneer announced an increasing sequence of prices. Prices called out at the auction would climb to the point where total demand was just below the issue size. At that point, the previously announced price would mark the single, market-clearing price that placed the entire issue.” p. B-24
Example: Nationwide Narrowband Auction
July 1994

- Five large licenses
- Can win at most three
- Simultaneous ascending-bid auction
  - Analogous to uniform price auction
    (all five sold for $80,000,000)

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Efficiency: What if we set uniform price equal to $85, what are incentives?

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Profit
38  28  18  84
Uniform-price auction
Price = 75, Revenue = 375

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AirTouch
V₁ = 75
V₂ = 5
V₃ = 3

SkyTel
V₁ = 85
V₂ = 65
V₃ = 7

What about seller revenues?

Price

Pay-Your-Bid

Uniform-Price

Vickrey

Residual Supply
Qₛ = ∑ᵢ≠₁ Qᵢ(p)

Demand
Qᵢ(p)

Quantity

p₀

Qᵢ(p₀)
Efficient auctions may yield high revenues

**Theorem.** With flat demands drawn independently from the same regular distribution, seller’s revenue is maximized by awarding good to those with highest values.

**Theorem.** With downward-sloping demands drawn independently from the same exponential distribution, seller’s revenue is maximized by awarding good to those with highest values.

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**Efficiency and revenue comparisons**

- Robust examples where
  
  Vickrey > Pay-Your-Bid > Uniform-Price

  in both efficiency and seller revenues

- Robust examples where
  
  Vickrey > Uniform-Price > Pay-Your-Bid

  in efficiency

- Vickrey is always best in efficiency

- Revenue results are ambiguous
Sealed-bid Versus Ascending

• Ascending: need to modify uniform-price rule to be an analog of Vickrey
  – “Ausubel Auction”

• Ascending Advantages
  – Overcomes Winner’s Curse
  – Price discovery: investment in considering relevant contingencies
  – Helps with efficiency, reduces bid shading
  – Revenue: bidders get sucked into competition (rationality??)

Sealed-bid Versus Ascending

• Sealed Bid Advantages
  – Simple to coordinate and run, bidders prepare bids once and at own convenience
    • No plans for what happens if internet goes down, etc.
  – Ex ante asymmetries
    • Important both for entry and bidding
  – Risk aversion (?)
    • Sealed bid dominates with private values, risk aversion
    • Result overturned with common values, agency problems
Sealed-bid Versus Ascending

• Sealed Bid Advantages cont’d
  – Avoid collusion
    • Ascending auction for gives bidders a chance to communicate, negotiate, retaliate
    • Especially severe with multiple objects
  – Sealed bid collusion
    • Requires either relatively frequent interaction in similar auctions, or
    • Meeting in a “smoke-filled room”

Conclusion

• Ascending better if…
  – Lots of competition
  – Enough strong, fairly symmetric bidders
  – Need for price discovery
  – High enough stakes to justify time and expense
• Sealed bid if…
  – Want to encourage entry of small bidders
  – Relying on fringe of smaller bidders to extract revenue from large bidder
  – Concerns about tacit collusion