Common Value Auctions

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Market Design

Common Value Auctions and the “Winner’s Curse”

• The setting
  – Bidders are uncertain about true value of object to them
  – Information that other bidders have would influence their beliefs

• Examples
  – Oil
  – Timber
  – Items with resale value
Mineral Rights Model

• Model
  – Oil in the ground worth \( v \)
  – Bidders see signals \( x_i = v + e_i \)
  – Errors \( e_i \) have mean 0, independent

• The Winner’s Curse
  – What is expected value of oil given \( x_i \)?

  – What is expected value of oil given \( x_i \) AND \( x_i \) is highest of all signals?

Mineral Rights Model: Equilibrium Analysis

• Deriving an Equilibrium
  – Wilson, Milgrom & Weber
  – Bidders must shade their bids to account for winner’s curse, even in second-price auction

• Results
  – Ascending auctions allow bidders to learn, alleviate winner’s curse
  – Second-price auction makes price depend on other bidders’ information
  – Revenue equivalence breaks down
    • Ranking: Ascending, Second-price, First-price
**Equilibrium Bidding in Second Price Auction**

Let $Y_i = \max_{j \neq i} X_j$

Solve: $\max_b \Pr(\beta(Y_i) \leq b \mid X_i = x_i) \times$

$$\left( E[v \mid X_i = x_i, \beta(Y_i) \leq b] - E[\beta(Y_i) \mid X_i = x_i, \beta(Y_i) \leq b] \right)$$

$$= \int_0^{\beta^{-1}(b)} \left( E[v \mid X_i = x_i, Y_i = y_i] - \beta(y_i) \right) \cdot f(y_i \mid x_i) dy_i$$

Bid until integrand turns negative:

$E[v \mid X_i = x_i, Y_i = \beta^{-1}(b)] = \beta(\beta^{-1}(b))$

Letting $\beta(x_i) = b, \ E[v \mid X_i = x_i, Y_i = x_i] = \beta(x_i)$

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**Equilibrium Bidding in Second Price Auction: Interpretations**

$\beta(x_i) = E[v \mid X_i = x_i, Y_i = x_i]$

- Expected value, conditional on tying for winner with opponent of the same type
- Requires bidders to understand strategy of opponents
- Accounts for winner’s curse
  - Conditions on the event that my bid matters
Understanding the conditional expectation

\[ \beta(x_i) = E[v \mid X_i = x_i, Y_i = x_y] \]

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<th>The plot has oil</th>
<th>The plot does not have oil</th>
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Hendricks & Porter, 1988

Testing the theory of equilibrium bidding

- Development of game theory as language for modeling multi-person decision problems in 1980’s made this an important issue.
  - experimental evidence raised questions about the predictive power of game theory; but stakes were low, bidders lacked experience.
  - Capen, Clapp, & Campbell claimed that ex post returns on offshore oil and gas auctions were below-market because bidders suffered from the winner’s curse.
- Private value models are difficult to test with field data since bidder values are private.
- Their approach: focus on common value auctions and use data on ex post values of offshore oil and gas tracts to test the theory of bidding in first-price auctions.
Things to notice

- Develops a theory tailored to the empirical setting (but still has unrealistic assumptions)
- Derives a set of theoretical predictions and provides empirical evidence supporting them; when taken together, the results seem difficult to reconcile with alternative theories (what are alternatives?)
- Empirical setting with value information enables much more powerful tests than one with bidding data alone; direct evidence that theory has some bearing on reality
- Econometrics are simple but convincing
- This is a classic paper in the literature.

Drainage versus Wildcat Tracts

Wildcat Tracts
- Located in unexplored areas.
- Firms allowed to conduct seismic studies but not to drill
- Symmetric information structure: \( F(X_1, \ldots, X_N, V) \); signals are noisy, but equally informative about unknown, common value \( V \).

Drainage Tracts
- Adjacent to tracts where oil and gas deposits have been discovered.
- Asymmetric information structure: neighbor firm(s) have private drilling info about \( V \); non-neighbor firms do not.
- Model: one informed (neighbor) bidder sees \( H = \mathbb{E}[V | X, Z] \), \( N \) uninformed (non-neighbor) bidders see public signal \( Z \) such that \( \mathbb{E}[V | Z] > R \).
- Allow for asymmetries in tract value due to possible common pool problems: \( V \) for neighbor firm, \( V - c \) for non-neighbor firms.
Auction Mechanism

• Tracts are auctioned simultaneously in a sale: 100-200 tracts in wildcat sales, about 25 tracts per drainage sale.

• Auction is first-price sealed bid.

• Wildcat tract is about 5000 acres; drainage tracts are typically 2250 acres.

• Reserve price is $15 per acre on wildcat tracts; $25 per acre on drainage.

• Terms: 5 year term, automatically renewable if productive; 1/6 royalty rate.

Data

• Location and sale date of tracts sold in sales held between 1954-1970.

• Identity of bidders and amounts bid; if joint bid, identity of participants and their shares.

• Number, date, and depth of wells drilled.

• Monthly production through 1991 of oil, condensate, natural gas, and other hydrocarbons.

• Ex post values: converted production flows into revenues using real wellhead prices at date of sale (i.e., static expectations); deducted drilling costs based on annual API survey of drilling costs per foot; discounted returns at 5%. 
Empirical Strategy: Drainage Auctions

- Work out the equilibrium to the model; extreme case of “winner’s curse”.

- Derive predictions about returns to neighbor and non-neighbor bidders and distribution of their bids. Asymmetry makes the model rich in this respect.

- Determine whether the data are consistent with those predictions.

- To be convincing, model needs to deliver predictions that are difficult to reconcile with plausible alternatives.
Drainage Tracts: Equilibrium

Equilibrium determines distributions of informed and maximum uninformed bids → N=1.

Prop 1: Uninformed bidder (U) has to bid.

- If U does not participate, informed bidder (I) bids r; U’s best reply is to bid \( r + \varepsilon \) since \( E[H|Z] > R \).

Prop 2: U has to bid randomly.

- If U bids \( b > R \), then I’s best reply is to bid \( b + \varepsilon \) iff \( H > b \). But then U wins only when tract is unprofitable, i.e., \( H < b \) → strong dose of winner’s curse.

Prop 3: There exists an equilibrium in which U mixes over \( \{0\}, \{r, E[H] - c\} \) and I uses a pure strategy

\[
\alpha^*(h) = \begin{cases} 
E[H|H \leq h; z] - c & h > \hat{h} \\
R & \hat{h} \geq h \geq R \\
0 & \hat{h} < R 
\end{cases}
\]

where \( \hat{h} \) solves \( E[H|H < \hat{h}; z] - c = R \).

Testable Predictions

1. Neighbor firm earns positive expected profits. (Confirmed in the data.)
2. Non-neighbor firms earn zero expected profits; strictly negative conditional on neighbor firms not bidding. (Confirmed.)
3. Non-neighbor firms have a lower aggregate participation rate than neighbor firms. (Neighbors bid 87% of time; non-neighbors bid 57% of the time.)
4. Neighbor firms should have a higher win rate than non-neighbor firms (Neighbor firms win rate was 57%).
5. The bidding strategy of the neighbor firm is independent of the number of non-neighbor firms. (No effect of N on neighbor bids found in the regressions.)
6. The distribution of neighbor bids should have a mass point at \( r \) and coincide with the distribution of the maximum non-neighbor bid above \( r \). (NOT confirmed.)

- Note that the mapping from signals to bids is not the same, just the bid distribution.

However, model did not account for an important institutional feature: high bid was rejected approximately 20% of the time, typically when only one low bid is submitted.

6’. Random reserve price R eliminates mass point at \( r \); \( G_0(b) < G_1(b) \) for bids in support of R.
Assumptions and Alternatives

One informed bidder: 74 of the 124 tracts had more than one neighbor firm.

- 16 of the 74 tracts received more than one bid.
- Behavior and outcomes on multi-neighbor tracts were not significantly different than on single neighbor tracts.

These results suggest that neighbor firms did not compete.

Main alternative is asymmetries in cost rather than information.

- Production was unitized.
- Adverse selection effects - non-neighbor profits are negative when neighbors bid.