An Introduction to Sponsored Search Advertising

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Sponsored Search Auctions


- Hal Varian, Google chief economist:
  - “What most people don’t realize is that all that money comes pennies at a time.”

- Today we’ll discuss internet keyword auctions.
ergonomic keyboard

About 439,000 results (0.25 seconds)

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Ergonomic Keyboards
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Brands for ergonomic keyboard: Microsoft Logitech Apple Kinesis Relaim

Shopping results for ergonomic keyboard

Logitech Cordless Desktop Wave Pro Wireless Keyboard - English - US ★★★★★ 297 reviews - $58 new, $47 used - 63 stores
Microsoft Natural Ergonomic Keyboard 4000 Wired Keyboard - English - North ★★★★★ 1,239 reviews - $26 new, $22 used - 177 stores
Goldtouch Apple Compatible Keyboard Wired Keyboard - White $89 new - 74 stores

Natural Ergonomic Keyboard 4000
Learn about the key features of this ergonomic keyboard.
www.microsoft.com/?slanguage=en-us/mouseandkeyboard/ProductDetails.aspx?pid... - Cached

Ergonomic keyboard - Wikipedia, the free encyclopedia
An ergonomic keyboard is a computer keyboard designed with ergonomic considerations
Ergonomic keyboard - Wikipedia, the free encyclopedia
An ergonomic keyboard is a computer keyboard designed with ergonomic considerations to minimize muscle strain and a host of related problems.

en.wikipedia.org/wiki/Ergonomic_keyboard - Cached page
Keyword Auctions

- Advertiser submit bids for keywords
  - Offer a dollar payment *per click*.
  - Alternatives: price per impression, or per conversion.
- Separate auction for every query
  - Positions awarded in order of bid (more on this later).
  - Advertisers pay bid of the advertiser in the position below.
  - “Generalized second price” auction format.
- Some important features
  - Multiple positions, but advertisers submit only a single bid.
  - Search is highly targeted, and transaction oriented.
Brief History of Sponsored Search Auctions

- Pre-1994: advertising sold on a per-impression basis, traditional direct sales to advertisers.
- 1994: Overture (then GoTo) allows advertisers to bid for keywords, offering some amount per click. Advertisers pay their bids.
- Late 1990s: Yahoo! and MSN adopt Overture, but mechanism proves unstable - advertisers constantly change bids to avoid paying more than necessary.
- 2002: Google modifies keyword auction to have advertisers pay minimum amount necessary to maintain their position (i.e. GSP)-followed by Yahoo! and MSN.
- 2005: Yahoo! buys Overture, Microsoft develops own platform
- 2010: Yahoo! outsources search and search ads to Microsoft
Current Auction Format

- Real-time
- Pay-per-click
- Click weighted
- Generalized second price auction
Why a real-time auction?

- Real-time auction
  - Prices vary widely: reserve of .05 binds frequently, some phrases $30/click
  - Millions of search phrases: top 2 million are only about 80% of searches, long tail
  - Hundreds of thousands of advertisers
  - Small advertisers join system all the time
  - Time sensitive items
  - Changing products and profits

- Costs of real-time auction
  - Firms need to monitor and fine-tune
  - Limits ability to price discriminate
Why a pay-per click (PPC) auction? (1)

- Alternatives
  - Pay per “impression” – this is what is being sold
  - Pay per action – pay for conversions

- Pay per impression
  - Advertiser bears risk for traffic quality
  - Search engine has private information about bot traffic, etc.
  - Traffic quality on partner network especially uncertain
  - In long term, advertisers monitor performance and adjust bids, but short term risk from fluctuations
Why a pay-per click (PPC) auction? (2)

- Pay per action
  - Use cookies to track conversions
  - Allows firms to enter profit on each item to advertising platform and let platform optimize for you

- Pay per click
  - Minimizes risk for advertisers relative to pay per impression
  - Simpler for search engine, easy technology
Click-weighted auctions

- All three search engines use click-weighted auctions
  - Original design: rank firms by their bids
- Unweighted per-click auctions can lead to lower revenue
- Example: search for Paris
  - Paris, France travel
    - Profit $.50/click, click-through rate 5% = $.025
  - Paris Hilton sex videos
    - Profit $5/click, click-through rate .25% = $.0125
  - Paris Hilton sex videos outbids Paris, France travel
  - Paris Hilton ad generates half the revenue for search engine
- Advantages of unweighted auctions
  - Do not require search engine to estimate click-through rates
  - Advertisers don’t want unnecessary clicks, write accurate ad text
Pay-your-bid v. Generalized Second Price

- Pay-your-bid
  - Early auction designs used a pay-your-bid format
  - Outcome was cycling
  - Firms incrementally outbid one another prices so high that it is unprofitable to win; then drop bid dramatically
  - Leads to inefficient outcomes

- Generalized second price
  - All three platforms now use a variant of a second-price auction, where advertisers pay minimum required to maintain position
  - Nice stability properties
  - The search engines continue to update their rules
Fig. 1. “Sawtooth” bidding pattern. (a) 14 h. (b) 1 week.
Example: Pay-your-Bid

- Two positions: receive 200 and 100 clicks per day
- Advertisers 1,2,3 have per-click values $10, $4, $2.

- Overture auction (pay your bid)
  - Advertiser 3 will offer up to $2 per click
  - Advertiser 2 has to bid $2.01 to get second slot
  - Advertiser 1 wants to bid $2.02 to get top slot.
  - But then advertiser 2 wants to top this, and so on.

- Pay your bid auction is unstable!
Pricing and Equilibrium: A Motivating Example

- Two positions: receive 200 and 100 clicks per day
- Advertisers 1, 2, 3 have per-click values $10, $4, $2.

- Efficient allocation
  - Advertiser 1 gets top slot: value created $200 \times 10 = 2000$
  - Advertiser 2 gets 2nd slot: value created $100 \times 4 = 400$

- Total value creation: $2400$
Example: competitive eqm

- Two positions: receive 200 and 100 clicks per day
- Advertisers 1, 2, 3 have per-click values $10, $4, $2.

- Competitive equilibrium
  - Set prices for slots \((p_1, p_2)\) so that demand = supply

- Example: \(p_2 = 2\) and \(p_1 = 4\)
  - Advertiser 3 demands nothing
  - Advertiser 2 demands slot 2: \(100 \times (4-2) > 200 \times (4-4) = 0\)
  - Advertiser 1 demands slot 1: \(200 \times (10-4) > 100 \times (10-2)\)

- Efficient outcome with revenue: $800 + $200 = $1000
Example: competitive eqm

- Two positions: receive 200 and 100 clicks per day
- Advertisers 1, 2, 3 have per-click values $10, $4, $2.

- Many possible mkt-clearing prices \((p_1, p_2)\)
  - Advertiser 3 must demand nothing, so \(p_1, p_2 \geq 2\)
  - Advertiser 2 must demand slot 2, so
    - \(p_2 \leq 4\), and \(2p_1 - p_2 \geq 4\) (so that \(200*(4 - p_1) < 100*(4-p_2)\))
  - Advertiser 1 must demand slot 1, so
    - \(2p_1 - p_2 \leq 10\) (so that \(200*(10 - p_1) < 100*(10-p_2)\))

- Allocation efficient with revenue: \(200p_1 + 100p_2\)
Possible competitive equilibrium prices!

Can we use an auction to “find” these prices?
Vickrey Auction

- Bidders submit bids (price per click)
- Auctioneer treats bids as values
  - Finds allocation that maximizes value created
  - So high bid gets top slot, and so forth
- Vickrey payment for bidder $j$
  - Note that if bidder $j$ gets a slot, he is “displacing” other bidders by moving them down a slot.
  - Compute the lost value from this displacement (e.g. if $j$ pushes $k$ down a slot, $k$ loses clicks that are worth some amount to $k$)
  - Bidder $j$’s payment equals total “displacement” cost, or equivalently the *externality* $j$ imposes on other bidders.
Vickrey Auction, cont.

- Second price auction is a Vickrey auction
  - Winner “displaces” second highest bidder
  - Winner pays the displaced value: $2^{nd}$ high bid
  - Also a Google auction with one click for sale!

- General properties of Vickrey auction
  - Dominant strategy to bid truthfully (bid = value)!
  - Outcome is efficient (maximizes total value)!
Example: Vickrey auction

- Two positions: receive 200 and 100 clicks per day
- Advertisers 1, 2, 3 have per-click values $10, $4, $2.

- Vickrey auction
  - Advertisers are submit bids, assigned efficiently given submitted bids, and have to pay the value their ad displaces.
  - Dominant strategy to bid one’s true value.
Deriving the Vickrey prices

- Two positions: receive 200 and 100 clicks per day
- Advertisers 1, 2, 3 have per-click values $10, $4, $2.

- Vickrey payment for Bidder 2
  - Bidder 2 displaces 3 from slot 2
  - Value lost from displacing 3: $2 \times 100 = $200
  - So Bidder 2 must pay $200 (for 100 clicks), or $2 per click.

- Vickrey payment for Bidder 1
  - Displaces 3 from slot 2: must pay $200
  - Displaces 2 from slot 1 to 2: must pay $4 \times (200-100) = $400
  - So Bidder 1 must pay $600 (for 200 clicks), or $3 per click.

- Vickrey “prices” are therefore $p_2 = 2$ and $p_1 = 3$, revenue $800$. 
Vickrey prices are the lowest competitive equilibrium prices!
Google “GSP” Auction

- Generalized Second Price Auction
  - Bidders submit bids (per click)
  - Top bid gets slot 1, second bid gets slot 2, etc.
  - Each bidder pays the bid of the bidder below him.

- Do the bidders want to bid truthfully?
Truthful bidding?

- Not a dominant strategy to bid “truthfully”!
  - Two positions, with 200 and 100 clicks.
  - Consider bidder with value 10
  - Suppose competing bids are 4 and 8.
    - Bidding 10 wins top slot, pay 8: profit $200 \cdot 2 = 400$.
    - Bidding 5 wins next slot, pay 4: profit $100 \cdot 6 = 600$.
  - This is the same tradeoff we have seen before…
    - Rather buy lower quantity at a lower price per unit, than get higher quantity at a higher price per unit
  - If competing bids are 6 and 8, better to bid 10…
Example: GSP auction

- Two positions: receive 200 and 100 clicks per day
- Advertisers 1, 2, 3 have per-click values $10, $4, $2.

- Generalized Second Price Auction
  - In this case, it is an equilibrium to bid truthfully
  - Bidder 2 gets slot 2 and pays $2 per click (or $200)
  - Bidder 1 gets slot 1 and pays $4 per click (or $800)

- Efficient allocation, revenue is $1000 (> Vickrey!)

- Why an equilibrium?
  - Bidder 3 would have to bid/pay $2 to get slot 2 – not worth it.
  - Bidder 2 would have to bid/pay $10 to get slot 1 – not worth it.
  - Bidder 1 could bid/pay $2 and get slot 2, but also not worth it.
GSP equilibrium prices

GSP prices are also competitive equilibrium prices!

Not the only GSP equilibrium, however
Example: GSP auction

- Two positions: receive 200 and 100 clicks per day
- Advertisers 1, 2, 3 have per-click values $10, $4, $2.

- Another GSP equilibrium (with Vickrey prices!)
  - Bidder 3 bids $2 and gets nothing
  - Bidder 2 bids $3 and pays $2 per click for slot 2
  - Bidder 1 bids $10 and pays $3 per click for slot 1

- And another GSP equilibrium (w/ higher prices)
  - Bidder 3 bids $3 and gets nothing
  - Bidder 2 bids $5 and pays $3 per click for slot 2
  - Bidder 1 bids $6 and pays $5 per click for slot 1
GSP equilibrium prices are also competitive equilibrium prices!
General Model

- $K$ positions $k=1,\ldots,K$
- $N$ bidders $i = 1,\ldots,N$

- Bidder $i$ values position $k$ at $u_{ik} = v_n \cdot x_k$
  - $x_k$ is quantity of clicks, $x_1>x_2>\ldots>x_K$
  - $v_n$ is value of a click, $v_1>v_2>\ldots>v_K$

- Efficient allocation is assortative
  - Bidder $k$ should get slot $k$ to max total value.
GSP Auction Rules

- Each agent $i$ submits bid $b_i$
  - Interpret as “maximum amount $i$ offers to pay per click”
- Positions assigned in order of bids
- Agent $i$’s price per click is set equal to the bid of agent in the next slot down.
- Let $b^k$ denote $k$th highest value and $v^k$ value.
- Payoff of $k$th highest bidder:
  \[ v^k \cdot x_k - b^{k+1} \cdot x_k = (v^k - b^{k+1}) \cdot x_k \]
GSP equilibrium analysis

- Full information Nash equilibrium
  - NE means no bidder wants to change positions
- Nash eqm is a bid profile $b^1, \ldots, b^K$ such that
  \[
  (v^k - b^{k+1}) \cdot x_k \geq (v^k - b^{m+1}) \cdot x_m \quad \text{for } m > k
  \]
  \[
  (v^k - b^{k+1}) \cdot x_k \geq (v^k - b^{m}) \cdot x_m \quad \text{for } m < k
  \]
- Lots of Nash equilibria, including some that are inefficient…
**Locally Envy-Free**

- **Definition**: An equilibrium is *locally envy-free* if no player can improve his payoff by exchanging bids with the player ranked one position above him.

- Motivation: “squeezing” – if an equilibrium is not LEF, there might be an incentive to squeeze.

- Add the constraint for all $k$

  $$(v_k - b_{k+1}) \cdot x_k \geq (v_k - b_k) \cdot x_{k-1}$$
Stable Assignments

- Close connection between
  - GSP equilibria / Competitive eqm / Stable assignments!

- Think of bidders being “matched” to positions. Matching position $i$ to bidder $k$ with price $p_k$ gives
  - bidder payoff: $(v_i - p_k)x_k$ and position payoff $p_kx_k$

- Stability: no bidder/position want to block.
  - All stable assignments are efficient (assortative).
  - Relevant blocks are bidders looking to move up or down one position. (think about this).
Stable assignments

- At a stable assignment, matching is efficient.
- Each position $k$ commands a price $p_k$.

- Prices that support a stable allocation satisfy:
  \[(v_k - p_k) \cdot x_k \geq (v_k - p_{k-1}) \cdot x_{k-1}\]
  \[(v_k - p_k) \cdot x_k \geq (v_k - p_{k+1}) \cdot x_{k+1}\]

- These are the conditions for a competitive equilibria
  - Essentially they say that bidder $k$ demands position $k$
  - So therefore at these prices, supply = demand!
Equivalence Result

**Theorem:**

- Outcome of any locally envy-free equilibrium of the GSP is a stable assignment (i.e. competitive equilibrium allocation)
- Provided that $|N|>|K|$, any stable assignment (i.e. competitive equilibrium allocation) is an outcome of a locally envy-free equilibrium.
GSP locally envy-free equilibrium prices

The set of competitive equilibrium prices corresponds to the set of locally envy-free GSP equilibrium prices!
Revenue and Prices

- **Theorem**
  - There exists a bidder-optimal competitive equilibrium (equivalently, GSP equilibrium) and a seller-optimal one.
  - The bidder optimal competitive equilibrium is payoff-equivalent to the Vickrey outcome.

- **Corollary:** any locally envy free GSP equilibrium generates at least as much revenue as Vickrey.
Stable prices, generally

- Stable prices satisfy
  \[(v_k - p_k) \cdot x_k \geq (v_k - p_{k-1}) \cdot x_{k-1}\]
  \[(v_k - p_k) \cdot x_k \geq (v_k - p_{k+1}) \cdot x_{k+1}\]

- Re-arranging we get
  \[p_{k-1} x_{k-1} \geq p_k x_k + v_k (x_{k-1} - x_k)\]
  \[p_{k-1} x_{k-1} \leq p_k x_k + v_{k-1} (x_{k-1} - x_k)\]

- This gives us a relationship between price of one slot and the price of the slot above… can solve for stable prices from the bottom up…
Features of Equilibrium

- Allocation is efficient ( assortative )
- Increasing price of marginal clicks
  - Varian points out this is testable.
  - Implies bidders are click-constrained!
  - Pricing should be linear if bidders satiated…
- Bids “reveal” bounds on bidder values.
  - But this model is not realistic. We will return to a more realistic model…
Ascending auction

- What if there is incomplete information about values – does this change things, or is there a natural process through which market equilibrates?
- Suppose price rises from zero, advertisers can drop out at any time, fixing their bid.

- **Theorem (Edelman et al.).**
  - There is a unique perfect equilibrium in which advertisers drop out in order of their values.
  - The equilibrium is efficient and the prices are Vickrey.
  - The equilibrium is an ex post equilibrium – no one wants to go back and change their bidding after the auction ends.
Choices in Auction Design

- How many slots to sell?
  - Would search engine want to restrict slots available?
  - Could this ever increase revenue? Efficiency?

- Setting a reserve price?
  - What is the optimal reserve price?
  - Is it better to use a reserve price, or restrict slots?

- Clickability and “squashing”
  - What if ads have different “clickability”
  - Should you incorporate this in the auction? How?
Example: Slot Restriction

- Two positions with 200, 100 clicks
- Three bidders with values $2, $1, $1
- Should the seller sell only one position?
  - Focus on lowest equilibrium (Vickrey) prices.
  - Selling two positions: revenue of $300.
  - Selling one position: revenue of $200.
    - Bidder 2 and 3 will pay up to $1 per click.
    - Market clearing price is $1 per click for Bidder 1.
Example: Slot Restriction

- Two positions with 200, 100 clicks
- Three bidders with values $3, $3, $1
- Should the seller sell only one position?
  - Selling two positions: revenue of $500.
    - Bidder 2 will pay $1 /click for slot 2
    - Bidder 1 will pay $2 /click for slot 1
  - Selling one position: revenue of $600.
    - Bidder 2 will pay up to $3 per click for slot 1
    - Market clearing price is $3 per click for Bidder 1.
Example: Reserve Prices

- Two positions with 200, 100 clicks
- Three bidders with values $2, $1, $1

Can the seller benefit from a reserve price?

- No reserve price: revenue of $300.
- Reserve price of $2: revenue of $400
  - Sell only one position, but for $2 per click!
Example: Reserve Price

- Two positions with 200, 100 clicks
- Three bidders with values $3, $3, $1
- Can the seller benefit from a reserve price?
  - No reserve price: revenue of $500 (or $600 if sell just one position).
  - Reserve price of $3 (and sell both positions): revenue of $900!
- In general, is it better to use a reserve price, or to adjust the number of slots for sale?
Bidder-Specific Click Rates

- Some ads may be more relevant than others.
  - eg if query is “Pottery Barn,” what ad will get clicked?
- Extended model where click rates differ.
  - Suppose $\text{Pr}(\text{click}) = a_i \cdot x_k$
  - Values: $u_{ik} = v_i \cdot (a_i \cdot x_k) = (v_i a_i) x_k$
  - Bids are still made on a per-click basis
- Value rank: rank bids by expected revenue, by $b_i \cdot a_i$
  - Eqm allocation will maximize total value.
  - Bidder-optimal eqm will be payoff-equivalent to Vickrey
- Bid rank: rank bids directly by $b_i$.
  - May not be efficient, but may raise revenue.
“Squashing” Example

- Two positions with 200, 100 “base” clicks
- Three bidders
  - per-click values $2, $1, $1
  - click-thru rates: 2, 1, 1
- Rank bids by bid*CTR
  - Bidder 2 pays $1 per-click for position 2
  - Bidder 1 pays $0.50 per-click for position 1 (why?)
  - Total revenue: $1*100 + $0.50*400 = $300.
“Squashing” Example

- Two positions with 200, 100 “base” clicks
- Three bidders
  - per-click values $2, $1, $1
  - click-thru rates: 2, 1, 1
- Rank bids by bid (i.e. treat B1 “as if” CTR=1)
  - Bidder 2 pays $1 per-click for position 2
  - Bidder 1 pays $1 per-click for position 1. (why?)
  - Revenue: $1*100 + $1*400 = $500!
- Can squashing lead to inefficient matching of bidders and positions? When?
Issues not in basic model

- Is each query a separate competition?
  - Advertisers really have portfolio of bids & broad match…
  - They also have budget constraints, decreasing returns.
  - They also have a choice between competing platforms.

- Model doesn’t allow for much uncertainty
  - Click rates, effectiveness of advertising are known.
  - See Athey & Nekipelov…

- Many aspects of search not captured
  - The consumers are not in the basic model and so you cannot do welfare properly (Athey & Ellison)
  - How do people decide whether/what to click?
  - Is there an interaction with “organic” search?
  - “Broad match” and the use of algorithms… very important.