1 Collective Action: the Simple Case

A collective action situation is any situation in which some action affects (or at least has the potential to affect) a group collectively but must be provided by one or more individuals voluntarily. Prominent examples include voting in an election, reducing pollution, and contributing to a charitable cause. A collective action problem exists whenever the activity in question is individually costly. Take the example of reducing air pollution in a town with two steel factories, 1 and 2, each owned by a different person, a and b, respectively. Producing steel as cheaply as possible creates a large amount of air pollution, which affects everyone in the town, including individuals who do not own the factory producing the pollution. (Pollution is an example of a negative externality: a by-product of some other activity that affects individuals who do not control the activity in question.) Reducing pollution benefits all people to some degree, but the costs of reducing it (e.g., the scrubbers in the smoke stacks) are borne only by the owner. Furthermore, if a incurs these costs, but b does not, then pollution is reduced, but b gets to enjoy both the benefits of higher profits and cleaner air, unlike a, who enjoys cleaner air, but incurs lower profits. If the benefits from cleaner air are less than the difference between high profits and low profits, then a has no incentive to reduce pollution.

Formally, a collective action problem is often described in terms of “contribute” or “not,” where “contribute” describes taking the costly individual action that makes the group collectively better off, and – in the “two factory” case – is depicted as in Figure 1. The notation of the Figure is as follows: if exactly one of the two players contributes, then both players enjoy a benefit of \( B > 0 \) from the collective action (i.e., somewhat cleaner air). If both players contribute, then both players enjoy a “multiplied” benefit of \( kB > B \) (i.e., the collective benefit is multiplied by a factor of \( k > 1 \)). Any player who contributes incurs a private cost of \( C > 0 \). If neither player contributes, then each player receives a payoff that is normalized to equal 0.

As the figure indicates, a collective action problem exists whenever \((k - 1)B < C\) or \(B < C\). If \((k - 1)B < C\) and \(B < C\), then the only equilibrium outcome is that neither player contributes: regardless of what player a does, player b’s payoff is always higher when he or she does not contribute than if he or she does (in game theory terms, not contributing is a dominant strategy). This situation is troubling if the benefit from

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1Those familiar with game theory may recognize Figure 1 as encapsulating the famous prisoners’ dilemma.

2Note that this does not imply that both are equally “happy” with this outcome. Why?
contributing, \( B \), is also experienced by “players outside the game” (e.g., the people who do not own factories enjoy clean air). If \( B < C \) and \((k-1)B < C\), then a collective action problem exists in the sense that it is not clear which player will (or “should”) contribute. If player \( a \) contributes, then player \( b \) should not, and vice-versa. Accordingly, the players have an incentive to coordinate their actions, for otherwise it is possible that both will choose to not contribute. If \((k - 1)B > C \) but \( B < C \) (for example, suppose that \( k \) is huge), then there is also a collective action problem in a coordination sense: both players want to contribute so long as the other player contributes as well. If either player is reasonably uncertain about the other player’s decision, he or she may choose to “play it safe” and not contribute. (Of course, if \( B > C \) and \((k - 1)B > C \), then there is no collective action problem: both should (and presumably will) contribute, since they strictly benefit from contributing, regardless of what the other player does (analogous to the discussion of not contributing when \((k - 1)B < C \) and \( B < C \), contributing is a dominant strategy in this case.)

The situation in Figure 1 can be generalized to more than two players. Regardless of the number of actors, the main characteristic of a collective action problem is that the individual action in question is socially desirable and yet individually costly. Formally, a characterizing feature of such situations is that, while many or all individuals in the group want the action to be carried out (e.g., people want nuclear waste disposed of safely and securely), they would prefer that somebody else incur the cost of carrying it out (e.g., they want the nuclear waste to be safely and securely disposed of in somebody else’s “backyard”). Many social situations have a similarity to collective action problems, and one of the main goals of political economy is to understand the behavior and institutions that ameliorate or exacerbate the dilemmas caused by these situations.
A Collective Action Situation as a “Normal Form Game”

<table>
<thead>
<tr>
<th>Player a</th>
<th>Player b Contributes</th>
<th>Player b Doesn’t Contribute</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$u_a = kB - C$</td>
<td>$u_a = B - C$</td>
</tr>
<tr>
<td></td>
<td>$u_b = kB - C$</td>
<td>$u_b = B$</td>
</tr>
<tr>
<td></td>
<td>$u_a = B$</td>
<td>$u_a = 0$</td>
</tr>
<tr>
<td></td>
<td>$u_b = B - C$</td>
<td>$u_b = 0$</td>
</tr>
</tbody>
</table>

C: Cost of Contributing \( (C>0) \)
B: Benefit from One Contribution \( (B>0) \)
k: Multiplier for Two Contributions \( (k>1) \)

A Collective Action Problem Exists When \((k-1)B<C\)

Figure 1: Collective Action Game
1.1 Collective Action, More Broadly

In general, group decision making requires that the people in the group make *individual decisions* to participate in the collective decision making process. This participation can come in many forms – individuals may be asked to turnout and vote, draft legislation, investigate the potential values of different possible collective choices, among many other things. A useful way to conceptualize collective action is as a cooperation game between the members of the group, as discussed last class.

1.2 Threshold games

A *k*-threshold game with $n$ players is a simple way to formalize arguments about collective action. In a *k*-threshold game, a collective (or public) benefit is generated if at least $k$ players decide to individually *contribute* to the creation of the collective benefit. Each member who contributes is charged an individual cost of $C$. The benefit, if it is generated, is worth $B$ to each player, regardless of whether he or she contributed to the benefit’s creation. Each player $i$’s decision is denoted by $a_i \in \{0, 1\}$, with $a_i = 1$ denoting that player $i$ decided to contribute to the collective good and $a_i = 0$ otherwise. All players make their decisions simultaneously.

Since the players are all identical (they all have the same action sets, payoff functions, and information): the game is *symmetric*, we are interested in one question: *how many people contribute in equilibrium?* Throughout, suppose that $B > C > 0$. (The problem is trivial otherwise – what should happen in this case and why?)

$k = 1$. First, suppose that $k = 1$ – any one player can individually provide the benefit. It is clear that at least one person should contribute (any individual would be willing to do so) but *no more than one*. If person $i$ is contributing ($a_i = 1$), then person $j$ faces the following comparison: the payoff from contributing is $B - C$. The payoff from not contributing is $B$. Obviously, not contributing makes player $j$ better off than contributing. Thus, when $k = 1$, there is a unique equilibrium outcome: exactly one individual will contribute to the collective benefit.

$k > 1$. Suppose that $k > 1$. The logic from above shows that no more than $k$ people should contribute – otherwise, some people would be “wasting” their participation, costing themselves $C$ for no reason. Suppose that exactly $k - 1$ people contribute. Nobody is receiving the collective benefit, but one individual can exert effort and generate the benefit. For any individual $i$ with $a_i = 0$ in this scenario, the payoff from contributing is $B - C$ while the payoff from not contributing is $0$. Obviously, “one more person should step up” and contribute in this situation. Suppose that exactly $k$ individuals contribute – it is clear that none of the noncontributing individuals would want to contribute (this would be a waste) and that none of the contributing individuals would want to stop contributing (this would lose the collective benefit). Thus, this is an equilibrium. Suppose
that less than $k - 1$ individuals contribute. No non-contributor is individually willing to start contributing, and the contributors would prefer to stop contributing. Finally, it is clear that “nobody contributing” is an equilibrium. These are the only pure strategy Nash equilibrium outcomes. There are also mixed strategy Nash equilibria, but we will leave those aside for the moment, other than to note that these equilibria make all individuals who contribute with positive probability indifferent between contributing and not contributing: the expected payoffs of both actions are identical in such equilibria.

**Comparative Statics: The effects of $B$, $C$, $k$, and $n$.** Shepsle and Bonchek discuss the effects of the parameters of the model on the likelihood of contributing. The discussion is somewhat misleading, however, as a lot of the (very important) factors they discuss are not in the model. The “psychological pressures,” for example, are not in the theory in any sensible way. This does not imply that I do not believe such factors are important – to the contrary, I believe they are very important and therefore should be included in our theory.

If we consider only Nash equilibria of the game described above, $n$ and $k$ have no effect on the model’s predictions so long as $k > 1$. While we can tell an intuitively appealing story about why individual contributions should become more likely as $k$ approaches $n$, there are always exactly two pure strategy Nash equilibria: $k$ contributing and 0 contributing. Similarly, as long as $B > C > 0$, $B$ and $C$ have no effect on the predicted equilibrium outcomes.

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3See Question 1.
1.3 Collective Action as Coordination Games

The $k$-threshold game is a special case of a coordination problem. In particular, the group would like to be able to “pick” exactly $k$ individuals to contribute to the collective good and secure the collective benefit at the minimum feasible cost. Coordination games are very important in political economy. From simple examples like which side of the road to drive on to more complicated examples such as scheduling votes in a legislature, there are plenty of situations in which a group is interested in getting all (or at least most) of its membership “on the same page.”

**Pure Coordination: The Left/Right Game.** Suppose that person $i$ walking past person $j$ in the hall and they are going in opposite directions. Both players have a common interest – they both want to avoid hitting each other and they don’t care how we do this: there are two actions – go left ($L$) or go right ($R$) – and the players must choose simultaneously. If the players both choose the same action, they each receive a payoff of 1. Otherwise, they each receive a payoff of -1. There are two pure strategy Nash equilibria to this game – both go left ($L, L$) or both go right ($R, R$). This game is displayed in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>$a_j = L$</th>
<th>$a_j = R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_i = L$</td>
<td>$u_i = 1$</td>
<td>$u_j = 1$</td>
</tr>
<tr>
<td>$a_i = R$</td>
<td>$u_i = -1$</td>
<td>$u_j = -1$</td>
</tr>
</tbody>
</table>

Table 1: Pure Coordination: The Left/Right Game

**Anti-coordination: Matching Pennies.** Suppose that two players, $i$ and $j$, play the following game: each player takes a penny and simultaneously places the penny either “heads up” ($H$) or “tails up” ($T$). The players then reveal the pennies to each other – if the coins match, then player $i$ wins both of the pennies. Otherwise, player $j$ wins both. The players each receive a payoff of 1 if he or she wins the other person’s penny and -1 if he or she loses the penny to the other player. This is the purest example of an “anti-coordination” game: the players’ interests are diametrically opposed. This game is displayed in Table 2.

**Asymmetric Coordination.** Suppose that two people, Alice and Bob, both want to hang out together on Friday evening. Alice wants to watch “Team America: World

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4There is also a mixed strategy Nash equilibrium in which we both flip a coin to decide which direction to go. The pure strategy Nash equilibria are more efficient than the mixed one (why?), but the pure strategy Nash equilibria seem to require some sort of pre-play communication (perhaps through “norms,” such as “walk on the right hand side.”)
Police” (denoted by \( M \) for “movie”), while Bob wants to go to the Ashlee Simpson concert (denoted by \( C \) for “concert”). Both Alice and Bob prefer hanging out together to hanging out separately. The payoffs are displayed in Table 3. In this game, there is no “pure” desire for coordination, because the players have strict (and different) preferences over which type of coordination occurs. There are two pure strategy Nash equilibria and a mixed strategy Nash equilibrium to this game.

**Pareto-Ranked Coordination Games.** Suppose that, in the above example, we change Bob’s preferences so that he too strictly prefers “Team America: World Police” to Ashlee Simpson. This new game is displayed in Table 4. There are still 2 pure strategy Nash equilibria (though one is uniquely “the best” in terms of Pareto dominance) as well as a mixed strategy Nash equilibrium.

<table>
<thead>
<tr>
<th></th>
<th>( a_j = M )</th>
<th>( a_j = C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_i = M )</td>
<td>( u_i = 2 )</td>
<td>( u_i = 0 )</td>
</tr>
<tr>
<td></td>
<td>( u_j = 2 )</td>
<td>( u_j = 0 )</td>
</tr>
<tr>
<td>( a_i = C )</td>
<td>( u_i = 0 )</td>
<td>( u_i = 1 )</td>
</tr>
<tr>
<td></td>
<td>( u_j = 0 )</td>
<td>( u_j = 1 )</td>
</tr>
</tbody>
</table>

Table 4: Pareto-Ranked Coordination Game

2 Problems for Section 1

1. Consider the \( k \)-threshold game and discuss the reasons that producing the public might be easier or harder when \( k = n \) than achieving production when \( k < n \). Are
there any situations that your answer/intuition help explain group behavior and or institutions?

2. Consider the symmetric coordination game displayed in Table 1. Can you think of a (simple) way involving only one of the players that the players might use to solve the coordination problem? (Hint: think about one player “saying something” to the other player.) Would this work for the the asymmetric coordination game displayed in Table 3? Why or why not? Is this surprising? What other problem or problems might be involved in using this “coordination device” prior to playing the the asymmetric coordination game, as opposed to the symmetric one?

3. Discuss what would happen in a $k$-threshold game if the players made their choices sequentially instead of simultaneously. For concreteness, suppose that player 1 makes his or her decision first, player 2 makes his or her decision second, and so forth, through player $n$. Assume that each player gets to observe all the decisions that have been made prior to making his or her decision. Assuming that $B > C$ for each player, will the good be provided? Which players will provide it? (Hint: think about the case of $k = 1$ and the player $n$’s decision, and then “backward induct. The more general case of $k > 1$ is a little bit more complicated, but essentially equivalent. Think about player $n - k + 1$’s decision and its ramifications, given the incentives of player $n$ in the $k = 1$ case.)

3 Turnout

One of the most basic forms of collective action in politics is the choice to vote in an election, (i.e., “turnout”). Put simply, turnout is a threshold game, where the threshold itself is endogenously determined by the number of people who turnout to vote for the other candidate(s). The simplest case is a two candidate election between candidates $D$ and $R$. Denote the set of people who support candidate $j$ by $N_j$, and denote the number of people in $N_j$ (i.e., $|N_j|$) by $n_j$, for $j \in \{D, R\}$. For simplicity, and as usual, we will denote the set of all voters by $N \equiv N_D \cup N_R$. (Note that we can ignore people who don’t strictly prefer either candidate: they have no instrumental incentive to vote, since they don’t care which candidate wins.) Suppose that the election is simple plurality and denote the number of people voting for candidate $j$ by $V_j$. Candidate $D$ wins if $V_D > V_R$, candidate $R$ wins if $V_R > V_D$, and a fair coin toss is used to determine the winner if $V_R = V_D$.

The preferred candidate of voter $i$ is denoted by $\tau_i$ (so $\tau_i = D$ if $i \in N_D$ and $\tau_i = R$ if $i \in N_R$). Each voter $i$ simultaneously makes a choice, $v_i \in \{0, 1\}$, with $v_i = 1$ if he or she turns out (and casts a vote for his or her preferred candidate) and $v_i = 0$ if $i$ stays home and abstains from voting. Also, suppose that each voter $i$ receives a payoff of 1 if
his or her preferred candidate wins the election, a payoff of zero if the other candidate wins the election, and pays a cost of \(0 < c_i < 1\) if \(v_i = 1\). Then, considering the case of \(\tau_i = D\) (i.e., \(i \in N_D\)), since the case of \(\tau_i = R\) is symmetric, the payoff of player \(i\) is

\[
u_i(v_i; V_R, V_D, \tau_i = D, c_i) = \begin{cases} 
1 & \text{if } V_D > V_R \text{ and } v_i = 0 \\
\frac{1}{2} & \text{if } V_R = V_D \text{ and } v_i = 0 \\
0 & \text{if } V_D < V_R \text{ and } v_i = 0 \\
1 - c_i & \text{if } V_D > V_R \text{ and } v_i = 1 \\
\frac{1}{2} - c_i & \text{if } V_R = V_D \text{ and } v_i = 1 \\
-c_i & \text{if } V_D < V_R \text{ and } v_i = 1 
\end{cases}
\]

(1)

To understand how this is a slightly more complicated version of a threshold game, consider when voter \(i\) should and should not vote, given the other voters’ choices. Under the assumption that \(\tau_i = D\) (again, the case of \(\tau_i = R\) is symmetric), suppose that \(v_i = 1\) and \(V_D - V_R > 1\): the voter’s preferred candidate is winning by more than one vote with voter \(i\)’s vote. Can the voter increase his or her payoff by not voting? In this case, since \(V_D - 1 > V_R\) (by the supposition that \(V_D - V_R > 1\)), Equation (1) yields the following payoffs for voter \(i\) as a function of voting and abstaining:

\[
u_i(v_i = 1; V_R, V_D, \tau_i = D, c_i) = 1 - c_i, \text{ while} \\
u_i(v_i = 0; V_R, V_D - 1, \tau_i = D, c_i) = 1
\]

Thus, since \(c_i > 0\) (voting takes time and energy, after all) and voter \(i\)’s vote is not necessary for candidate \(D\) to win the election, voter \(i\) might as well stay at home.\(^5\)

Now consider the nearly symmetric case of \(v_i = 1\) and \(V_R - V_D > 0\): the voter’s preferred candidate is losing, even with voter \(i\)’s vote. Can the voter increase his or her payoff by not voting? In this case, since \(V_R > V_D\) (by the supposition that \(V_R - V_D > 0\)), Equation (1) yields the following payoffs for voter \(i\) as a function of voting and abstaining:

\[
u_i(v_i = 1; V_R, V_D, \tau_i = D, c_i) = -c_i, \text{ while} \\
u_i(v_i = 0; V_R, V_D - 1, \tau_i = D, c_i) = 0
\]

Again, since \(c_i > 0\) and voter \(i\)’s vote will not result in candidate \(D\) winning the election, voter \(i\) might as well stay at home.

The interesting cases are:

1. \(v_i = 1\) and \(V_D = V_R + 1\): in this case, voter \(i\)’s vote is necessary for \(D\) to win outright. Equation (1) yields the following payoffs for voter \(i\) as a function of voting and abstaining:

\[
u_i(v_i = 1; V_R, V_D, \tau_i = D, c_i) = 1 - c_i, \text{ while} \\
u_i(v_i = 0; V_R, V_D - 1, \tau_i = D, c_i) = \frac{1}{2}
\]

\(^5\)Of course, there are reasons to vote other than affecting the outcome. These reasons are most easily incorporated in the model by changing the value of \(c_i\) – perhaps even allowing it to be negative, implying that not voting is costly (perhaps in psychological or social terms).
In this case, should voter \( i \) turn out and vote (i.e., is \( u_i(v_i = 1; V_R, V_D, \tau_i = D, c_i) > u_i(v_i = 0; V_R, V_D, \tau_i = D, c_i) \))? The answer depends on \( c_i \). If \( c_i < \frac{1}{2} \), then voter \( i \) should vote, since the cost of voting is less than the policy benefit of deciding the election. However, if \( c_i > \frac{1}{2} \), then voter \( i \) should abstain, even though his or her vote would be pivotal in electing candidate \( D \). (In the highly unlikely case of \( c_i = \frac{1}{2} \), the voter is indifferent between voting and abstaining.)

2. \( v_i = 1 \) and \( V_D = V_R \): in this case, voter \( i \)'s vote is necessary for \( D \) to be tied with \( R \) (with a coin toss being used to break the tie). Equation (1) yields the following payoffs for voter \( i \) as a function of voting and abstaining:

\[
\begin{align*}
  u_i(v_i = 1; V_R, V_D, \tau_i = D, c_i) &= \frac{1}{2} - c_i, \\
  u_i(v_i = 0; V_R, V_D = 1, \tau_i = D, c_i) &= 0
\end{align*}
\]

In this case, should voter \( i \) turn out and vote (i.e., is \( u_i(v_i = 1; V_R, V_D, \tau_i = D, c_i) > u_i(v_i = 0; V_R, V_D, \tau_i = D, c_i) \))? The answer depends on \( c_i \). If \( c_i < \frac{1}{2} \), then voter \( i \) should vote, since the cost of voting is less than the policy benefit of deciding the election. However, if \( c_i > \frac{1}{2} \), then voter \( i \) should abstain, even though his or her vote would be pivotal in electing candidate \( D \). (In the highly unlikely case of \( c_i = \frac{1}{2} \), the voter is indifferent between voting and abstaining.)

Notice that the only cases where a voter with \( c_i > 0 \) would ever want to cast a vote are when that voter’s vote will either make or break a tie between the two candidates. Also, notice that voters with \( \frac{1}{2} < c_i < 1 \) should never vote, even though the benefit from their preferred candidate winning is strictly larger than \( c_i \). This is because any one voter can only make or break a tie – his or her vote will not turn a “sure loss” for their candidate into a “sure win.”

Suppose now that \( 0 < c_i < \frac{1}{2} \) for all voters \( i \). The turnout game is very similar to a threshold game in the sense that each voter in \( N_D \) wants to vote if and only if either (1) \( V_D = V_R \) or (2) \( V_D = V_R + 1 \) with voter \( i \)'s vote. Put another way, voter \( i \) wants to turn out and vote for his or her preferred candidate if and only if the preferred candidate is trailing by one vote or tied with the opposing candidate without voter \( i \)'s vote. Since the same logic holds for voters who prefer \( R \) (with the labels switched, of course), it “turns out” that the turnout game is a “two-sided” threshold game in which the thresholds are themselves endogenously determined (i.e., the threshold for the provision of the public good among the voters in \( N_R \) (namely, the election of \( R \)) is determined by the strategic behaviors of the voters in \( N_D \)).

\footnote{Notice also that this conclusion depends on the tie-breaking rule being a coin flip. If, on the other hand, the tie-breaking rule was, say, \( D \) wins when \( V_D = V_R \) (or, equivalently, if \( R \) wins when \( V_R = V_D \), then voters with \( \frac{1}{2} < c_i < 1 \) will want to cast a vote in one of the two above cases – which of the two cases it is for any given voter \( i \) depends on whether voter \( i \)'s favored candidate is the one that wins or loses a tie vote.)}
Definition 1  A pure strategy Nash equilibrium of the turnout game is a vector of decisions by the voters, \( v^* = \{v^*_i\}_{i \in N} \) such that, for each voter \( i \), voter \( i \) is maximizing his or her payoff \( u_i \), as defined in Equation 1, given the other voters’ decisions. In words, a pure strategy Nash equilibrium in this game is a list of zeros and ones such that each voter with \( v^*_i = 1 \) would receive a lower payoff if he or she chose \( v'_i = 0 \), and any voter \( i \) with \( v^*_i = 0 \) would receive a lower payoff if he or she choose \( v'_i = 1 \).

3.0.1 Turnout with Incomplete Information: Riker and Ordeshook

In real-world elections, it is unrealistic to expect that people play a pure strategy Nash equilibrium on election day. There are several reasons for this, but perhaps the most powerful assumption (implicitly) underlying the above analysis is that the voters have complete information about each others’ preferences (i.e., all voters know \( \tau \equiv \{\tau_i\}_{i \in N} \)) and their costs of voting (i.e., all voters know \( c \equiv \{c_i\}_{i \in N} \)). What if, instead, an individual simply had a belief about the probability that his or her vote will be pivotal in a two-candidate election (i.e., the probability that his or her vote will end up “making or breaking a tie” between the two candidates)? Riker and Ordeshook [1968] first considered this question,\(^7\) and derived a simple and direct calculation of the “net benefit” from voting. Denoting the direct cost of voting by \( c_i \geq 0 \), as above, any psychological benefits from voting (sometimes referred to as the “duty term”) by \( d_i \geq 0 \), the benefit from having the preferred candidate win the election by \( b_i \), and the probability that \( i \)’s vote makes or breaks a tie (referred to as the pivot probability) by \( p_i \), Riker and Ordeshook show that the net benefit to \( i \) from voting (as opposed to abstaining), \( r_i \), is

\[
    r_i = p_i b_i - c_i + d_i. \tag{2}
\]

If \( r_i \) is positive, then voter \( i \) should vote, if \( r_i \) is negative, then \( i \) should abstain. Riker and Ordeshook [1968] and many others have argued that \( p_i \) is typically quite small – typically on the order of \( \frac{1}{n_V} \), where \( n_V \) is the number of people who vote, so something like \( \frac{1}{100,000,000} \) in a presidential election.\(^8\)

4 Problems for Section 3

1. Rewrite Equation 1 for the case where the tie-breaking rule is as follows: if \( V_D = V_R \), then candidate \( D \) wins. When would a voter want to cast a vote (in terms of \( V_R \) and \( V_D \)) under this rule is \( \tau_i = D \). What if \( \tau_i = R \)?

\(^7\)Palfrey and Rosenthal [1985] later considered it in more detail.

\(^8\)A caveat about applying this to presidential elections in the United States: such a “calculation” of \( p_i \) does not take into account the Electoral College. However, taking this into account is very complicated and does not change the result very much.
2. Suppose that \( c_i < \frac{1}{2} \) for all voters \( i \) and that \( n_D > n_R \). In a pure strategy Nash equilibrium, which candidate (or candidates) can win? Why? Describe the intuition behind your answer.

3. Do you vote? Do you vote in all elections (including local elections, primary elections, referenda, etc.) Why or why not? Describe how the following factors affect your choice to vote:
   
   (a) Your belief about how close the election will be.
   (b) The number of candidates in the election.
   (c) The amount of campaign literature you receive and/or campaign advertisements you see.
   (d) The “tone” of the campaign.
   (e) The office that the campaign is for (President, Senator, City Council, Dog Catcher, etc.).
   (f) The location of the polling place, the weather on election day, etc.

5  Cooperation

In many situations, universally cooperative behavior within a group leads to higher payoffs for all individuals than universally noncooperative behavior, yet cooperation cannot be sustained? Why? The reason often is that, conditional on everyone else in the group cooperating, any individual has an incentive to not cooperate (“defect”). These situations are called social dilemmas (we talked about these briefly before in the course). Consider a situation where two neighbors (Players A and B) each border a marsh. If the marsh is drained, both players will benefit from a reduction in swamp gas and mosquitoes. However, draining the marsh is costly.\(^9\) If we consider this problem as a game in which each of the two neighbors must choose whether to drain the marsh or do nothing, an example of this game’s normal form is shown in Table 5. The unique Nash equilibrium

<table>
<thead>
<tr>
<th>Player B</th>
<th>Drain</th>
<th>Do Nothing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Player A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Drain</td>
<td>1,1</td>
<td>-1,2</td>
</tr>
<tr>
<td>Do Nothing</td>
<td>2,-1</td>
<td>0,0</td>
</tr>
</tbody>
</table>

Table 5: Example of a Social Dilemma

\(^9\)The “drain the marsh” example is drawn from Chapter 8 of Shepsle and Bonchek [1997]. There are many other examples of social dilemmas, of course.
of the game pictured in Table 5 is for both players to “do nothing,” even though they both do better if they work together to drain the marsh. Indeed, this equilibrium is supported by “dominant” actions, referred to in game theory as dominant strategies.

**Definition 2** An action \( a \) is strictly dominant for a player \( i \) if, regardless of what the other players do, the payoff received by player \( i \) from taking action \( a \) is strictly higher than she would have received from any other action.

**Definition 3** An action \( a \) is weakly dominant for a player \( i \) if, regardless of what the other players do, the payoff received by player \( i \) from taking action \( a \) is at least as great as the payoff she would have received from any other action.

### 5.1 Possible Solutions to the Paradox of Cooperation

**Repeated Play.** Reputation and/or punishment might support cooperation in social dilemmas (SDs). Repetition does not solve it unless the number of times the game will be repeated is possibly infinite.

**Changing the Game: Altering Payoffs.** Mafia example.

<table>
<thead>
<tr>
<th>Player A</th>
<th>Player B</th>
<th>Drain</th>
<th>Do Nothing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drain</td>
<td>1,1</td>
<td>-1,-( \beta )</td>
<td></td>
</tr>
<tr>
<td>Do Nothing</td>
<td>-( \beta ),-1</td>
<td>0,0</td>
<td></td>
</tr>
</tbody>
</table>

Table 6: Modified Social Dilemma: Mafia Case

**Changing the Game: Third Party Enforcement.** In a SD, the participants all have an incentive to hire an outside enforcer, who essentially will receive a portion of the gains from cooperation in return for punishing any defectors. (This is the rationale behind Hobbes’s Leviathan.)

**Costly Enforcement.** How much does verification of cooperation and potential punishment of defectors cost? What if defecting is sometimes involuntary – what is the proper degree of punishment?

**Imperfect Enforcement.** Can the enforcer verify that cooperation occurred? Is the enforcer biased?

**Enforcer Incentives.** Can the enforcer be trusted not to “change the game” in some way that suits his or her interests?
6 Problems for Section 5

1. In the marsh draining example, verify that “do nothing” is a strictly dominant action.

References

