Adaptive Integration

Things to look for:

a) How does the code keep track of the sub intervals?
b) How does the code measure the errors?
c) How does it dole out permission to err?
d) Two computations are made for each interval. What is done with the results?

% Tim Sauer
function sum = adapquad(f,a0,b0,tol0)
sum=0; n=1; a(1)=a0; b(1)=b0; tol(1)=tol0; app(1)=trap(f,a,b);
while n > 0 % n is current position at end of the list
    c = (a(n)+b(n))/2; oldapp=app(n);
    app(n)=trap(f,a(n),c);app(n+1) = trap(f,c,b(n));
    if abs(oldapp-(app(n)+app(n+1))) < 3*tol(n)
        sum = sum + app(n) + app(n+1); % success
        n=n-1; % done with interval
    else % divide into two intervals
        b(n+1)=b(n); b(n)=c; % set up new intervals
        a(n+1)=c;
        tol(n)=tol(n)/2; tol(n+1) = tol(n);
        n=n+1; % go to end of list, repeat
    end
end

function s=trap(f,a,b)
s = (f(a)+f(b))*(b-a)/2;

Compare to the version below: what improvements have been made?
% Cleve Moler
% ...
% Recursive call
[Q,k] = quadtxstep(F, a, b, tol, fa, fc, fb, varargin{:});
fcount = k + 3;
% function [Q,fcount] = quadtxstep(F,a,b,tol,fa,fc,fb,varargin)

function [Q,fcount] = quadtxstep(F,a,b,tol,fa,fc,fb,varargin)
h = b - a;
c = (a + b)/2;
fd = F((a+c)/2,varargin{:});
fe = F((c+b)/2,varargin{:});
Q1 = h/6 * (fa + 4*fc + fb);
Q2 = h/12 * (fa + 4*fd + 2*fc + 4*fe + fb);
if abs(Q2 - Q1) <= tol
    Q = Q2 + (Q2 - Q1)/15;
    fcount = 2;
else
    [Qa,ka] = quadtxstep(F, a, c, tol, fa, fd, fc, varargin{:});
    [Qb,kb] = quadtxstep(F, c, b, tol, fc, fe, fb, varargin{:});
    Q = Qa + Qb;
    fcount = ka + kb + 2;
end

Guassian Quadrature

Define the inner product of function $f$ and $g$ on the interval $[−1,1]$ as

$$<f,g> \overset{\text{def}}{=} \int_{-1}^{1} f(x)g(x)dx$$

For the abscissa we use the zeros of the Legendre orthogonal polynomials

$p_0(x) = 1$
$p_1(x) = x$
$p_2(x) = x^2 - \frac{1}{3}$
$p_3(x) = x^3 - \frac{3}{5}x$

... 

Weights are the areas under the interpolating polynomials.
The two point form: $(x_0, x_1) = (±\frac{1}{\sqrt{3}})$ and $(c_0, c_1) = (1, 1)$
The three point form: $(x_0, x_1, x_2) = (-\frac{\sqrt{3}}{\sqrt{5}}, 0, \frac{\sqrt{3}}{\sqrt{5}})$ and $(c_0, c_1, c_2) = (\frac{5}{6}, \frac{8}{5}, \frac{5}{6})$