Prediction Markets

Market Design, Fall 2010
Logistics

- Friday, November 19 – Former students Scott Kominers and Judd Kessler; concluding comments from Al
- Friday, November 26 – No class (Thanksgiving)
- Friday, December 3 – Student project presentations. Volunteer!
- Sunday, December 12 – Submit projects to Ranjan Ahuja rahuja[at]hbs.edu. No extensions!
Outline

1. Prediction Markets Overview + Types of Contracts
2. Prediction Market Design
3. Information Aggregation
4. Corporate Prediction Markets / Crowdcast
Question: What is the purpose of the stock market?
Prediction Markets

- incentives for *truthful revelation*
- incentives for *research and information discovery*
- mechanism for *aggregating opinions*
A Typical Contract

Contract payoffs depend on unknown future events
Example: contract pays off $1 if $Y$ occurs by time $T$, 0 otherwise.

Event realizations must be specific.

- **Good:** The Patriots win Super Bowl XLV
- **Bad:** Tom Brady gets injured in November.

Warning! Seemingly well-defined contracts can have holes.
Winner-Take-All Contract:

- Pays out $1 if a specific event occurs, 0 otherwise.
- Market price $p$ for this contract gives the “market’s expectation” of the probability that this event will occur.

Why?

- Continuum of bettors
- Each better has beliefs about event E
- Finding: Equilibrium price yields bounds on mean beliefs


- Extended Manski model to include risk aversion
- Finding: With logarithmic utility, equilibrium price = mean beliefs.
Winner-Take-All Contract: Example

The Saddam Security

- Paid out $100 if Saddam ousted from power by June 2003.
- Traded on website tradesports.com
- Price on Jan 1, 2003: $55
- Price on March 1, 2003: $70
Index Contracts

Contracts pay out the value of a specific future event.

Example:
Contract pays $1 for each percentage point of the 2012 popular vote won by Sarah Palin.

Market prices of index contracts measure means.
Let \( y \) be Palin’s percentage of the popular vote.
Market price of contract reveals \( E[y] \).
Spread Contracts

- Amount of bet is fixed.
- Market trades based on cutoffs that determine whether event occurs

Example:
Contract pays even money if Sarah Palin wins more than y% of the popular vote.
Market trades based on y (I’ll buy 10 units at y = 43)
Market price reveals median value for y. (This is a fair bet if event is as likely to occur as not)
Carefully constructed markets can reveal information about the distribution of an uncertain future event:

- index contract 1 pays $y^2$
- index contract 2 pays $y$

Market prices reveal $E[y^2]$ and $E[y]$.

In general, contracts can be constructed to provide any desired order statistic about distributions.
Contingent Contracts

Cleverly designed contracts can predict contingent events.
Example: “Electability”
Candidates / Party vote share
How are Contracts Traded?

In many prediction markets, the mechanism used is the *continuous double auction*.

- This is the mechanism used on the NYSE.
The Iowa Electronic exchange trades contracts that pay off on mutually exclusive events.

A set of mutually exclusive events is called a *basket* (or a *bundle*.) IEM never loses money!

To enter the market, you may either

- Go long on a position
- Exchange $1 for a basket of goods.
Market Design Choice: IEM Model

Drawbacks:

- Two ways to make the same bet. (if you hold positions) Arbitrage?
- Hard to take a short position. Must first buy a basket.
- Your cash is held by IEM - no interest.
Market Design Choice: “InTrade” Model

To bet against an event, “short” the asset.
Suppose an asset that pays off $1 if $y$ occurs is trading at price $p$.

Shorting

- gives you $p$ dollars now...
- ...but you are “on the hook” for $1 if $y$ occurs. That is, $1 of assets are frozen in your account to make sure you can cover your bet.

Like margin trading.
Accuracy of Prediction Markets

1. Hollywood stock exchange: Oscar predictions
2. Iowa Electronic Exchange vs Gallup
3. HP Printer Sales vs internal sales prediction methods
4. Google internal prediction markets

But...an open question.
Prediction Market Challenge 1: Real vs Play Money

Question: Does Money Matter?
Real vs Play Money: Empirical Finding


- 2003 NFL season (14 games)
- Comparison of two websites:
  - TradeSports: Real $$
  - NewsFutures SportsExchange, in partnership with *USA Today*: Play $$

Findings:

- Neither site systematically out predicted the other
- Predictions from both sites were remarkably accurate
  - Authors entered aggregate opinions into a third sports betting site (which used play money.)
  - TradeSports and NewsFutures took 11th and 14th place out of 2000 experts, respectively.
Unlawful Internet Gambling Funding Prohibition Act (2003)

AN ACT To prevent the use of certain bank instruments for unlawful Internet gambling, and for other purposes.

The Congress finds as follows:

1. Internet gambling is primarily funded through personal use of bank instruments, including credit cards and wire transfers.

2. The National Gambling Impact Study Commission (1999) recommended legislation to prohibit wire transfers to Internet gambling sites or the banks which represent them.

3. Internet gambling is a major cause of debt collection problems for insured depository insts. & the consumer credit industry.

4. Internet gambling conducted through offshore jurisdictions has been identified by United States law enforcement officials as a significant money laundering vulnerability.
Prediction markets are markets for contracts that yield payments based on the outcome of an uncertain future event, such as a presidential election. Using these markets as forecasting tools could substantially improve decision making in the private and public sectors.

We argue that U.S. regulators should lower barriers to the creation and design of prediction markets by creating a safe harbor for certain types of small stakes markets. We believe our proposed change has the potential to stimulate innovation in the design and use of prediction markets throughout the economy, and in the process to provide information that will benefit the private sector and government alike.
Statement on Prediction Markets (coauthors)

Prediction Market Challenge 2: Thin Markets

Thin betting on events like

- Who will be the next pope?
- Who will be the next nobel prize winner in economics?

Why is it risky to place a limit order in a thin market?

⇒ On tradesports, limit orders are free.

Why is it risky to make any trade in a thin market?
A proper scoring rule is a method for soliciting a probability distribution from an individual.

Let $X$ be a random variable that takes on $K$ possible states (indexed by $i$).

Let $s_i(\cdot)$ be a mapping that describes a state contingent payoff from a reported probability distribution $r$ for $X$.

$s_i(\cdot)$ is a proper scoring rule if it is incentive compatible for an individual with beliefs $p$ to report beliefs truthfully. That is,

$$p = \arg \max_r \left[ \sum_i p_i s_i(r) \right]$$

and

$$\sum_i p_i s_i(p) \geq 0.$$
Examples of Proper Scoring Rules

Quadratic Scoring Rule (Brier, 1950)

\[ s_i(r) = a_i + b \left( 2r_i - \sum_j r_j^2 \right). \]

Logarithmic Scoring Rule (Good, 1952)

\[ s_i(r) = a_i + b \log(r_i). \]
Market Scoring Rules

Robin Hanson (2003) insight: Let market participants correct each others’ probabilities!

Market for variable $X$ which can take on $K$ states.

Marketplace ‘sponsor’ chooses initial distribution $q$.

Market participants may adjust the distribution to $r$.

Each user makes payment (upon realization of the event) to the previous user who has made an adjustment:

$$c_i = s_i(r^t) - s_i(r^{t-1}).$$
Market Scoring Rules

\[ c_i = s_i(r^t) - s_i(r^{t-1}). \]

- Market sponsor *subsidizes* the market
- Subsidy is capped
Market Scoring Rules as Automated Market Maker

How to adapt market scoring rules to the notion of buying and selling assets?
Insight: Let users adjust s!

- For each asset $i$, $s_i(p)$ represents ‘shares outstanding.’
- Bettor names quantity $\bar{s}_i$ she would like to buy or sell; asks how much?
- New prices would given by $p^{-1}(s + \bar{s})$
- Amount quoted for bundle given by integral of prices over quantity range.
Market Scoring Rules

\[ c_i = s_i(r^t) - s_i(r^{t-1}). \]

- Market sponsor subsidizes market; Subsidy is capped
- Has ‘feel’ of buying and selling assets.
- Can be operated alongside traditional prediction market, where assets “payout of 1 when state is i” are traded.
- Quoted prices? Probabilities represent prices for very small trades
- Predominant market design in internal corporate prediction markets.

**Open Question**: How well do market scoring rules aggregate information? (See e.g. Ostrovsky 2009)
Applications

1. HP Printer sales
2. Avian Bird flu
3. Google Prediction Markets (HBS Case)
4. Cambrian House (HBS Case)
5. Crowdcast (including drug prediction markets)
Open Research Questions

Theory:

- How well do various markets aggregate information?
- What role does a ‘balanced budget’ (or capped subsidy) play in limiting market mechanism?
- What happens with prediction markets are taken seriously to make decisions? (feedback loop)

Empirical:

- Data from Crowdcast, other sources.
- How many users do we need for accurate predictions?
- When can we expect accurate predictions? (type of market, type of event, type of user etc)
No-Trade Theorem Logic

- Aumann (1976), Milgrom-Stokey (1982)
- No trade amongst partially informed traders

Solution?

- Naive Traders
- Noise Traders
- Non-Strategic Traders
- Market Scoring Rules
Ostrovsky (2009)

Logic:

- Sequence of partially informed traders make adjustments to probabilities; payoffs according to MSR
- Players are strategic:
  - Consider history of adjustments
  - Consider impact of adjustments on future prices
- Consider Perfect Bayesian Eqm:
  - Do equilibrium prices aggregate all available information?
Finding:
Yes, provided the security is *separable* (given an information structure).

A security is separable if, roughly, for any prior distribution of beliefs, there is at least one informed trader.

Example of non-separable security: (see Ostrovsky 2009)
Market Design 2010 Prediction Market