Comparison of two kinds of growth

<table>
<thead>
<tr>
<th>Linear Growth</th>
<th>Exponential Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Characterized by a constant number.</td>
<td>Characterized by a constant percent.</td>
</tr>
<tr>
<td>This constant growth rate is the <strong>slope</strong>.</td>
<td>This constant percent is the <strong>Growth Rate</strong>.</td>
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<tr>
<td>In linear growth we ADD this amount each time.</td>
<td>In exponential growth we MULTIPLY by a growth factor each time.</td>
</tr>
<tr>
<td>Equation looks like y = (growth rate)x + initial amount</td>
<td>Equation looks like y = initial amount(growth factor)^t</td>
</tr>
<tr>
<td>General Linear Equation y = f(x) = mx + b</td>
<td>General Exponential Equation y = a(1 ±r )^t Let (1 ± r) = b</td>
</tr>
</tbody>
</table>

**Difference between Growth Rate and Growth Factor.**

**WHY A GROWTH FACTOR**

If something grows by a percent, e.g. 5%, that percent is called the 'Rate' of growth.

However, when we want to increase by 5%, we multiply by 1.05, i.e. 100% + 5%. This sum is called the 'Growth Factor.'

If we decrease by a percent, say 8%, we multiply by 100% – 8% = 92%

This difference becomes our growth factor, also called a called a ‘decay’ factor.

In general:

**Growth Factor = 100% ± the Growth Rate**
**Finding Formulas for Exponential Equations**

**RATIO METHOD**

**General Formula for an Exponential Equation**

\[ y = f(x) = ab^x \]

where \( a \) is the *initial* amount i.e. when \( x=0 \)

and \( b \) is the Growth FACTOR if \( b>1 \) or

Decay FACTOR if \( 0 < b < 1 \).

**RATIO METHOD:** From two points: \((x_1, y_1)\) and \((x_2, y_2)\)

\[
\frac{ab^{x_2}}{ab^{x_1}} = \frac{y_2}{y_1}
\]

Notice the ‘\( a \)’ cancels

**Example:** Two points on a graph: \((-2, 128)\) and \((2, \frac{1}{2})\) (Decreasing)

**Step 1.** Find the Growth Factor, \( b \), by taking the ratio of two equations as shown below.

Use the general equation and substitute values given.

** Helpful hint:** Put the term with the larger exponent in the numerator.

\[
\frac{ab^2}{ab^{-2}} = \frac{1/2}{128} = \frac{1}{256}
\]

\[ b^4 = \frac{1}{256} \]

**Step 2:** Now find ‘\( b \)’, the growth FACTOR using the rules of exponents.

\[
\left(b^4\right)^{1/4} = \frac{1}{256^{1/4}}
\]

\[ b = \frac{1}{4} \]

This is a decay factor since \( b < 1 \)

So far, our formula is \( y = a(1/4)^x \)

**Step 3:** Find ‘\( a \)’, the initial value, by substituting back one of the points you used.

\[ 128 = a(1/4)^{-2} \]

\[ 128 = 16a \]

\[ a = 8 \]

**FORMULA becomes:** \( y = 8\left(\frac{1}{4}\right)^x \)