The Lucas Model

Emmanuel Farhi

Harvard

October 22, 2008
Poducers

- Many goods, indexed by $i$
- Technology
  $$Q_i = L_i$$
- Preferences
  $$U(C_i, L_i) = C_i - \frac{1}{\gamma} L_i^\gamma$$
Supply for each good

- Prices:
  - price of good \( i \), \( P_i \)
  - aggregate price level \( P \)

- Household labor supply

\[
\max C_i - \frac{1}{\gamma} L_i^\gamma
\]

s.t.

\[
PC_i = P_i L_i
\]

- Resulting labor supply function

\[
Q_i = L_i = \left( \frac{P_i}{P} \right)^{1/(\gamma-1)} \quad \text{or} \quad q_i = l_i = \frac{1}{\gamma-1} (p_i - p)
\]
Demand for each good

- Assume log-linear demand

\[ q_i = y + z_i - \eta (p_i - p) \]

- \( y \) aggregate real income, \( z_i \) shock to demand, \( \eta \) elasticity of demand for each good

\[ p = \bar{p}_i \text{ and } \bar{z}_i = 0 \]

- Microfoundations:
  - CES demand functions if \( \eta = 1 \)
  - \( \eta > 1 \) → different aggregation formula for \( p \)
  - approximation for \( y \)

- Cash in advance constraint

\[ C_i \leq M_i / P \]

Aggregate demand

\[ y = m - p \]
Aggregate supply and aggregate demand

- Aggregate supply
  \[ y = \bar{q}_i \]

- Aggregate demand from cash in advance constraint (\( C_i \leq M_i / P \) revenues from money issuance rebated lump sum)
  \[ y = m - p \]
Market clearing for good $i$

\[ \frac{p_i - p}{\gamma - 1} = y + z_i - \eta (p_i - p) \iff p_i = \frac{\gamma - 1}{1 + \eta \gamma - \eta} (y + z_i) + p \]

\[ \implies \text{averaging} \]

\[ p = \frac{\gamma - 1}{1 + \eta \gamma - \eta} y + p \iff y = 0 \]

Market clearing for money (aggregate demand)

\[ m = p \]

Money is neutral
Imperfect information

- Price of good $i$
  \[ p_i = p + p_i - p = p + r_i \]

- Individual does not observe $r_i$, must estimate it from $p_i$ only (producer/shoper) \( \rightarrow E[r_i|p_i] \) with rational expectations

- Signal extraction problem (high $p_i$ signals high $m$ or high $z_i$)

- Assumption, producer sets (certainty equivalent not consistent with utility maximization)
  \[ l_i = \frac{1}{\gamma - 1} E[r_i|p_i] \]

- Assume monetary shock $m$ and demand shocks $z_i$ normally distributed, independent \( \implies p, p_i \) and $r_i$ normal
Signal extraction and Lucas supply curve

- Signal extraction

\[ E[r_i|p_i] = \frac{V_r}{V_r + V_p} (p_i - E[p]) \]

- Labor supply

\[ l_i = \frac{1}{\gamma - 1} \frac{V_r}{V_r + V_p} (p_i - E[p]) \equiv b (p_i - E[p]) \]

- Lucas supply curve

\[ y = b (p - E[p]) \]

- Inflation augmented Philipps curve vs. Philipps curve
Equilibrium: \( p \) and \( y \)

- Lucas supply curve (aggregate supply)
  \[ y = b (p - E[p]) \]

- Aggregate demand
  \[ y = m - p \]

- Equilibrium
  \[ p = E[m] + \frac{1}{1 + b} (m - E[m]) \]
  \[ y = \frac{b}{1 + b} (m - E[m]) \]

- Aggregate price \( p \) lognormal, \( E[p] = E[m] \)
  \[ V_p = V_m / (1 + b)^2 \]
Using demand and supply for good $i$, solve for $r_i = p_i - p$

$$l_i = b (p - E[p]) + z_i - \eta (p_i - p) \text{ and } l_i = b (p_i - p) + b (p - E[p])$$

$$\implies r_i = p_i - p = \frac{z_i}{\eta + b}$$

Relative price $r_i$ lognormal $E[r_i] = 0$ and $V_r = V_z / (\eta + b)^2$
Equilibrium: $b$

- **Equation for $b$**

\[
b = \frac{1}{\gamma - 1} \frac{V_z}{V_z + \left(\frac{\eta + b}{1 + b}\right)^2 V_m}
\]

\[\implies b = \frac{1}{\gamma - 1} \frac{V_z}{V_z + V_m} \text{ if } \eta = 1\]

- **Summing up**

\[
y = \frac{b}{1 + b} \left( m - E[m] \right) \text{ and } p = E[m] + \frac{1}{1 + b} \left( m - E[m] \right)
\]

- **Unexpected money shocks not neutral:**
  - increase in $p$ less than for one
  - increase in $y$
  - key: only un-anticipated increases in money supply!
Phillips curve and Lucas critique

- Suppose $m_t$ random walk plus drift
  
  $$m_t = m_{t-1} + c + u_t \text{ with } u_t \text{ white noise}$$

- Equilibrium price and inflation
  
  $$p_t = m_{t-1} + c + \frac{1}{1+b} u_t \text{ and } \pi_t = c + \frac{b}{1+b} u_{t-1} + \frac{1}{1+b} u_t$$

- Output
  
  $$y_t = \frac{b}{1+b} u_t$$

- Philips curve but not exploitable tradeoff $\pi, y$

- Lucas critique
Monetary policy can stabilize output only if policymakers have more information than agents.

Any portion of policy that responds to publicly available info (unemployment...) irrelevant to real economy

\[ m = m^* + v \]

Not good justification of stabilization: just make information available to public.
International evidence on output-inflation tradeoffs

- Idea
  \[ y = \frac{b}{1 + b} (m - E[m]) \text{ where } b \left( V_m \right) \]

- Use
  - log nominal GDP: unit-elastic demand curve \( m = p + y \)
  - log nominal GDP change unpredictable \( \Delta (p + y)_t = a + u_t \) with \( u_t \) white noise

- Regression
  \[
  y_{it} = c_i + \gamma_i t + \tau_i \Delta (p + y)_{it} + \lambda y_{it-1} + \varepsilon_{yit}
  \]
  \[
  \tau = \alpha + \beta \sigma_{\Delta(p+y),i} + \varepsilon_{i\tau}
  \]

- Result (43 countries cf. Ball, Makiw, Romer) \( \beta = 1.639 \) (0.482) with \( R^2 = 0.201 \)
Difficulties

- Evidence that publicly announced changes in monetary policy have real effects
- To generate large employment fluctuation, requires large labor supply elasticity (as for RBC models)
- Imperfect information?