The Shape of Temptation: Implications for the Economic Lives of the Poor*

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1 Introduction

One of the more robust stylized facts about the poor is that they borrow at very high interest rates. Aleem (1990) in a survey of money lenders finds an average interest rate of 78.5% per year. Even formal micro-financial institutions charge extremely high rates. In Mexico, for example, prominent MFIs charge 90%+ per year. Such high rates are remarkably common. Existing models have tried to understand what supply side could produce such high rates, be it transaction costs, monopoly power or credit market failures induced by asymmetry of information. We instead ask the demand side question: why would the poor want to borrow at such high rates?

We can understand this in traditional models through the Euler equation:

$$U'(c_t) \geq \delta R U'(c_{t+1})$$

If someone is borrowing at rate R, then marginal utility today must be higher than marginal utility tomorrow scaled by R and the discount factor. This is because the person could always borrow less, thereby generating R more units of consumption tomorrow. Note this is true irrespective of credit constraints, savings constraints or other market failures. Some of this borrowing occurs only at times when marginal utility of consumption is particularly high: for example, a loan to deal with a health shock. But much borrowing is repeated and not simply the result of a shock, such as that needed to finance working capital. For example, (farmers typically borrow at high rates and Karlan and Mullainathan (2008) find street vendors borrowing for many years at upwards of 5% day to finance their working capital.

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To understand such repeated, steady state borrowing, the traditional model would rely on one of two forces.[1] First, marginal utility in the future could be particularly low, thereby offsetting the high R on the right hand side of the equation. For example, individuals may borrow to finance very high return investments which raise income and consumption significantly tomorrow. Put differently, this implies the poor who borrow at very high rates should soon be much less poor. The second possibility is that discount factors could be very low, that borrowers are very myopic. Since the notion that the poor are soon to be non-poor appears counterfactual—high interest rate borrowing has been observed across time and space, in many places where the poor stay poor—the traditional model can only explain high interest rate borrowing through high discount rates.[2]

In this paper, we produce a model to provide an alternative explanation. This model also fits several other stylized facts, which allow it to be clearly differentiated from a high discount rate model. The starting point of the model is the emerging literature on time inconsistency (Shefrin and Thaler, Laibson, O'Donaghue and Rabin), wherein individuals can display both myopic behavior (for short-horizon decisions) and far-sighted behavior (for long-horizon decisions). Our model modifies existing hyperbolic discounting models in one important way. Current models assume that time inconsistency arises because present selves are, in the moment, tempted to consume more than the long-run self would have wanted. Here, we generalize to assume consumption has two components: x_t and z_t. The first, x_t, reflects consumption on which there is no temptation. Both long-run and short-run selves value this consumption. The second, z_t, reflects consumption where there is temptation–the utility the consumer gets from z is only valued by the self that is taking the decision—future selves do not value the fact that you have given into a temptation. This simple generalization of the hyperbolic model turns out to be extremely useful: providing a way to analyze how the extent of temptation may change with income or make predictions about specific consumption items rather than consumption as a whole.[3]

[1] Steady state borrowing is necessary for without it, one could easily understand high interest rate borrowing: it occurs when $u(c_t)$ happens to be very high today.

[2] Complications to the model also produce other candidate explanations but each of these is also counter-factual in some way. For example, one could argue that the poor cannot cut back on consumption because they are against some sort of "minimum consumption" constraint. This, however, appears implausible given the extreme volatility of consumption observed (cites). At the least, the poor could consume less in the high consumption states. One could also argue that a Stone-Geary utility function is more appropriate: that the poor face significant probability of death and need to consume today to reduce it. This runs counter to the trends in mortality rates for poor populations in most parts of the world. Finally, one could argue that the realized interest rates are not the rates actually paid if default rates are high. In fact, default rates are very low (for example, in MFIs).

[3] We do not take a position on whether this model is a more accurate description or simply a reformulation. On the one hand, the idea that temptation is not simply about more today versus tomorrow but is embodied in specific goods is intuitive. On the other hand, such good-specific temptations could be captured in a hyperbolic model by allowing goods to deliver different types of consumption at different points in time (as in Gruber and Koszegi for cigarettes). In short, we have chosen our model because it provides a more tractable language

The core technical result from this model which makes it tractable is that we can derive a modified Euler equation to describe consumption. In the traditional Euler equation individuals would equate marginal utility today ($U'(x)$) with the expected discount-rate and interest-rate adjusted marginal utility tomorrow ($\delta rU(x)$). In the temptation-modified Euler equation, the marginal utility tomorrow is further adjusted by a "temptation tax". Individuals know that a dollar transferred to tomorrow would be partly spent on x goods and partly spent on z goods. Since from the vantage point of today, only the x goods "matter", the z consumption can be viewed as a tax. So marginal utility today is equated to $\delta RU'(x))[1 - z'(c)]$ where $z'(c)$ is the marginal propensity to consume z. This temptation tax is the core of our intuition. In effect, individuals discount future income and consumption (if the choice of x and z is left to future selves) by $\delta[1 - z'(c)]$. This also provides our explanation of high interest rate borrowing: individuals will borrow in steady state if they feel that a fraction of their future income is in any case going to be “wasted”.

This framework provides us a language to express our key assumption: the function $z(x)$ is concave.[4] Here, $z(x)$ represents how much will be spent on temptation goods given the spending on regular goods. A concave $z(x)$ implies that as individuals spend more and more, a smaller fraction of total spending will go towards temptation. For example, this implies that if a poor person living on dollar a day spends \$.25 on temptation goods, then a person spending \$2 a day above \$.25 but below \$.50. In particular, this does not imply that the rich are not spending more on temptations, merely that they're not spending proportionally more. We discussion possible justifications of this assumption below: One could imagine that temptations are primitive in nature, representing such basic temptations as fat, sugar, alcohol, etc. If, as a practical matter, fancy donuts can only get so expensive (and even then only gain very little in temptingness), the resulting $z(x)$ function would be concave. Moreover, it is easy to imagine that temptations are spur of the minute purchases whereas most expensive goods require planning and foresight to even purchase (e.g. a very expensive car). Of course, in the end this assumption can be judged as any assumption is, by the quality of predictions it permits.

The first results follow rather directly from the assumption. The poor face a steep $z'(c)$ and therefore be less willing to save. They realize that any marginal dollar they save will likely be spent on temptation goods that they do not save. Wealthier individuals are on a flatter part of $z(x)$ and therefore are in a better position to save. Hence the poor will consume more of their income today. This has implications for apparent discount rates. Suppose that consumption data generated by the model were used to estimate discount rates using the traditional Euler equation; in other words, as if the true model had no temptation goods, only total consumption c. Then, even if the only source of heterogeneity

for thinking about essential concepts such as how the extent of temptation varies.

[4]\Below we show that for any $z(x)$ function we can reconstruct a $U()$ and $V()$ which generates that function. The key in this model is that for a given level of spending in a time period, the split of that spending between x and z is given solely by $U'(x) = V'(z)$ without consideration to the intertemporal optimization. Hence one can speak of $z(x)$ as a primitive.

between people is income, estimate discount factors would decline systematically with income. In other words, the poor would look like they discounted the future more, even though they do not. This result is direct: the poor, even at the same discount factor, face a higher temptation tax and hence would consume more of their income today and appear more myopic. While this result is a direct consequence of concavity of $z(x)$, it illustrates the analytical rationale for separating z and x goods, rather than working with a quasi-hyperbolic model. We can directly relate an intuitive assumption in the framework—concavity of $z(x)$—to an often observed phenomena—the apparent declining of discount factors with income.[5]

The fact that the temptation tax declines with income generates a behavioral poverty trap. There will exist a critical wealth level $\underline{w}$ such that the savings rate exhibits a discontinuous jump $\underline{w}$. In other words, individuals just below $\underline{w}$ will save very little or dissave, while those just above will save a lot more. Note that this poverty trap arises even though we have no lumpy investments or even credit constraints, the usual ingredients of poverty trap models. The intuition is a generalization of declining temptation taxes. Individuals face an added incentive to save since if they increase wealth they lower the temptation tax. On the other hand, at low enough wealth, this added incentive is offset by simply the level of the temptation tax. Put another way, the moderately well-off can save in the hopes of being sufficiently wealthy to avoid a large tax. The poor cannot save enough (or in a multi-period it would take long enough) to accomplish this; hence they simply dis-save. In short, people are present-biased because they are poor, but that in turn keeps them poor. In other words the impatience that the poor often shown is as much a result of their poverty as it is a cause.

Several other results follow from this framework which help us make sense of numerous stylized facts about the financial lives of the poor. First, declining temptation alters individual responses to uncertainty. An increase in variance can actually reduce savings. This is true even if the utility function (both $u(x)$ and $v(z)$) exhibit prudence. Second, investment choices change significantly. Without declining temptation, individuals? investments are determined by returns. With it, maximum scale (the total amount investable) also plays a role: a high return but small investment is not particularly attractive. This, we feel, is important to help parse recent studies on rates of return on investment. We argue this can help explain phenomena such as why individuals fail to undertake several very high return investments, even ones that are not lumpy such as fertilizer (Duflo, Kremer and Robinson) or small working capital (Lee, Kremer and Robinson). Relatedly, we also show that declining temptation places more structure on when debt can be bad from a self-control point of view. Access to many small loans (such as with a credit card) may exaggerate the self-control problem more than access to a loan which has a large minimum size requirement. Finally, we illustrate how monopolistic money-lenders would be able to create debt-traps for the poor, preventing them from adopting high-return technolo-

[5] In the economics literature, this has been remarked on as early as Irving Fisher (1932).

gies. This is in contrast to traditional models, where debt traps are not feasible, as discussed by Srinivasan (). This produces a behavioral resolution to the dispute on the feudal economy (Bhaduri, Srinivasan). While these results are qualitative, in the concluding section we discuss potentially quantitative tests of the model.

2 Model

2.1 Utility functions

The standard hyperbolic discounting model assumes a utility function of the type $u(c_0)+\beta\sum_t \delta^t u(c_t)$. As noted earlier, we deviate from this model for several reasons of analytic convenience. First, we are interested in temptation goods *per se*, in making predictions about goods which generate more time inconsistency than others. Some of our empirical predictions will be about this. In the hyperbolic model, to generate between-good variability in temptation requires modeling explicitly the inter-temporal utility dependencies of the goods (e.g. cigarettes). Second, we are interested in predictions below we are interested in understanding how temptations may vary with the level of income. This turns out to be a complex exercise in the hyperbolic model (see Appendix). In the model below this captured by an simple condition: are non-temptation goods a luxury good? Finally, as a practical matter the framework below allows us to work in two periods and the hyperbolic model shows no time inconsistency with only two periods. Note these arguments are of analytical convenience. The key contribution of this paper is to apply the notion of temptation to long-standing issues in development, not to offer an account of temptation that is different from the hyperbolic model.

To capture these ideas, suppose there are n goods consumed in each of two time periods.[6] Denote the spending on good $i = \{1, 2.....m, m + 1, ...n\}$ in time $t = \{1, 2\}$ by x_i^t. At time 2 each good provides separable utility so that individuals maximize:

$$E\{\sum_{i=1}^{n} W_i(x_i^2)\} \tag{1}$$

where the W_i are increasing, concave functions.

At time 1, we assume individuals have decision utility equal to:

$$\sum_{i=1}^{n} W_i(x_i^1) + \delta E\{\sum_{i=1}^{m} W_i(x_i^2)\} \tag{2}$$

where $E\{\cdot\}$ is the expectations operator defined in terms of the uncertainty in the model, as defined below. To understand this utility it is useful to contrast

[6] This setup can be obviously extended to more periods.

equation (**??**) with what would happen in a traditional discounted utility model:

$$\sum_{i=1}^{n} W_i(x_i^1) + \delta E\{\sum_{i=1}^{m} W_i(x_i^2)\} + E\{\sum_{i=m+1}^{n} W_i(x_i^2)\}$$

The key difference with our formulation is the omission of the term $E\{\sum_{i=m+1}^{n} W_i(x_i^2)\}$. Thus for goods $i = 1, .., m$ time 1 self uses the same utility function as time 2 self but merely discounts it by δ. But places no *no* weight on the utility derives from goods $i = m+1, ..., n$. Thus goods $m+1, ..., n$, are the goods which the future self values but which today's self does not put weight on.[7] We will refer to these goods as "temptation goods".

To simplify this, define

$$\begin{aligned} U_i(\cdot) &= W_i(\cdot) \text{ for } i \leq m \\ & \quad \textit{and} \\ V_i &= W_i(\cdot) \text{ for } i > m. \end{aligned}$$

We can then rewrite the above expression for expected life-time utility as

$$\sum_{i=1}^{m} U_i(x_i^1) + \sum_{i=m+1}^{n} V_i(x_i^1) + \delta E\{\sum_{i=1}^{m} U_i(x_i^2)\}$$

A further simplification is possible if we makes use of within period optimization. Choosing units so that all good prices are 1, we define

$$U(x^t) = \max_{x_1 \ldots x_m} \sum_{i=1}^{m} U_i(x_i^t), \sum_{i}^{m} x_i^t = x^t, t = 1, 2$$

and

$$V(z^t) = \max_{x_{m+1} \ldots x_n} \sum_{i=m+1}^{n} U_i(x_i^t), \sum_{i=m+1}^{n} x_i^t = z^t,$$

The key insight is that the total amount spent on any subset of goods must be spent optimally within that subset, even when the consumer is actually deciding over a larer set. We can therefore write the above maximand in the compact form

$$U(x^1) + V(z^1) + \delta E\{U(x^2)\}. \tag{3}$$

For the majority of this paper, this is the utility function we will use. In essence x denotes the index of spending on non-temptation goods, while z denotes the index of spending on temptation goods, much in the same way that

[7] A more general formulation would endow goods with a variable weight but for our purposes this simple model where there are goods with full and goods with no weight is particularly convenient.

c denotes the spending on total consumption in traditional and hyperbolic discounting models.[8] That both U and V should be increasing and concave follows from the corresponding assumptions about the W_i. Futhermore we will assume in everything we do that U is at least three times differentiable everywhere. We refrain from making the corresponding assumption for the V function to accomodate certain special cases.

2.1.1 Relation to hyperbolic discounting

As noted earlier, this model shares many features with a hyperbolic discounting model. In this particular context, however, these similarities are hard to make precise since in two periods, the hyerbolic discounting model is indistinguishable from an exponential discounting model. Time inconsistency in the hyperbolic discounting models comes from the fact that "next period's self" puts too little weight on the subsequent "self" from the point of view of today's self—therefore when there is just the next period's self nothing interesting happens.

To see the similarities, therefore, consider a T-period extension of our model where we ignore the role of effort. Under assumptions that gave us 3 the intertemporal maximand for this case can be written in the form

$$U(x^1) + V(z^1) + \sum_{t=1}^{T} \delta^t E\{U(x^t)\}$$

Maximizing $U(x) + V(z)$ subject to a budget constraint $x + z = c$, and the conditions $(x \geq 0, z \geq 0)$ gives us a function $x(c)$.

$$\begin{aligned} \widetilde{U}(c) &= U(x(c)) \\ & \quad and \\ \widetilde{V}(c) &= V(z(c)) \end{aligned}$$

Then the above expression becomes

$$\widetilde{U}(c^1) + \widetilde{V}(c^1) + \sum_{t=2}^{T} \delta^t E\{\widetilde{U}(c^t)\}$$

When $\widetilde{V}(c) = \alpha \widetilde{U}(c)$, this is the standard β-δ utlity function with $\beta = \frac{1}{1+\alpha}$. One case where this condition holds is when

$$\begin{aligned} U(x_t) &= \frac{x_t^{1-\alpha}}{1-\alpha} \text{ and} \\ V(z_t) &= A\frac{z_t^{1-\alpha}}{1-\alpha}. \end{aligned}$$

[8] As a result, there are several underlying good-specific utility functions that can generate the index utility described in 3. Thus while the separable utility formulation in 2 generates it, other more complicated utility functions can also give rise to it.

In other words two essentially identical CRRA functions: Under these assumptions, within period choice between x and z will give us $z_t = qx_t$ where $q = A^{\frac{1}{\alpha}}$. Substituing this into our maximand gives us

$$\begin{aligned} & \frac{x_0^{1-\alpha}}{1-\alpha} + \frac{(q)^{1-\alpha} x_0^{1-\alpha}}{1-\alpha} + \sum \delta^t \frac{x_t^{1-\alpha}}{1-\alpha} \\ = \quad & (1+q^{1-\alpha})[\frac{x_0^{1-\alpha}}{1-\alpha} + \sum \beta \delta^t \frac{x_t^{1-\alpha}}{1-\alpha}] \end{aligned}$$

mbb ./where $\beta = \frac{1}{1+q^{1-\alpha}}$, which is exactly in the hyperbolic form.

Of course more generally the models will be distinct, since they provide a different language for understanding temptations. Couched in our framework, there is no reason to assume that temptation utility, $V(\cdot)$, is proportional to non-temptation utility $U(\cdot)$. In fact, as we will see the ability to talk about the relative curvatures of $V(\cdot)$ and $U(\cdot)$ is essential to our analysis.

2.2 Maximization

Individuals maximize this utility subject to their budget constraint. In the first period they earn a deterministic "labor" income y^1. In that period they also have the opportunity to take a loan. which they can use to make an investment or simply consume more than they currently have. Let the amount of investment be k^1, and let $w^1 = y^1 - x^1 - z^1$ be his savings. Now if $w^1 \geq k^1$ then he is a net lender to the market. Otherwise he is a net borrower. Now define a credit supply function as follows: Let $r(w^1, k^1)$ be the interest rate he pays per unit of his net borrowing from the market $(k^1 - w^1)$. We assume that this function is defined both for positive and negative values of $k^1 - w^1$: when $k^1 - w^1$ is negative, this is interest he earns on his lending to the market. Also let $F^1(k^1, \theta)$ represent the intertemporal "production" technology that the person has access to which is a function of the total amount invested (k^1) and some potential shock (θ) that we assume is realized before the investment decisions are made but after savings is chosen. We impose the assumption that $F(k^1, \theta)$ is differentiable, increasing and (weakly) concave in k^1 and that $r(k^1, w^1)$ is differentiable, increasing and (weakly) convex in k^1 and differentiable, decreasing and concave in w^1 to rule out poverty traps that result from non-convexities in production (as in Galor-Zeira (1993)) or in the credit supply function (as in Banerjee-Newman (1994)).

Now we can define a resource generation function $f(w^1, \theta)$:

$$f(w^1, \theta) = \max_{k^1 \geq 0} \{F(k^1, \theta) - r(w^1, k^1)(k^1 - w^1)\} \tag{4}$$

The assumptions made above together imply that $f(w^1, \theta)$ is differentiable, increasing and (weakly) concave in w^1.

Note that this definition encompasses a number of very different cases: One is the case of perfect capital markets–$r(w^1, k^1)$ is a constant. A second is where there is only borrowing for consumption: This is the case when $F(k^1, \theta) \equiv 0$. In this case $k^1 = 0$ is clearly optimal and therefore all borrowing is for consumption. Finally we can choose the $r(w^1, k^1)$ function to approximate a curve that becomes vertical at some fixed value of k^1 for each value of w^1. This is the case of a credit limit.

In the second period the person also gets a potentially uncertain "labor" income $y^2(\theta')$. We assume that θ and θ' are independent random variables and that θ' is realized in the second ∪period before consumption decisions are taken

TIME LINE HERE

As defined above, maximizing $U(x^2) + V(z^2)$ subject to a budget constraint $x^2+z^2 = c^2$, and the conditions $(x^2 \geq 0, z^2 \geq 0)$ gives us functions $x^2(c^2), z^2(c^2)$. Under the standard assumption that both U and V is strictly concave, $x^2(c^2)$ and $z^2(c^2)$ will be non-decreasing in c^2. If V is also differentiable (in addition to U being differentiable), then $x^2(c^2)$ and $z^2(c^2)$ will be differentiable and strictly increasing everywhere except perhaps where the non-negativity constraint binds. Using the fact that $c^2 = f(w^1, \theta) + y^2(\theta')$, we can write this as $x^2(f(w^2, \theta) + y^2(\theta'))$. Also for future use, define $z^2(x^2)$ to be the function that is defined by the first order condition for maximizing $U(x^2) + V(z^2)$ subject to a budget constraint $x^2 + z^2 = c^2$, i.e by the equation $V'(z^2) = U'(x^2)$ and $W(c)$ to be the indirect utility function defined by maximizing $U(x)+V(z)$ subject to a budget constraint $x + z = c$, and the conditions $(x \geq 0, z \geq 0)$. Since both U and V are increasing and strictly concave, so is $W(c)$.

The decision-maker in the first period is assumed to be sophisticated and therefore takes this function into account in making his first period choices. We assume that in the first period he gets an income/endowment y^1

Therefore in the first period the decision problem is to maximize

$$U(x^1) + V(z^1) + \delta E_{\theta,\theta'}\{U(x^2(f^1(w^1, \theta) + y^2(\theta')))\}$$

subject to

$$w^1 = y^1 - x^1 - z^1$$

and

$$x^1 \geq 0, z^1 \geq 0.$$

2.3 First order conditions

If an interior optimum exists and $\frac{dx^2(f^2(w^2,\theta))}{df^2(w^2,\theta)}$ and $\frac{dV(z^1)}{dz^1}$ exists at the optimum, then the folloiwng conditons must hold.

$$\begin{aligned} \frac{dU(x^1)}{dx^1} &= \lambda \\ \frac{dV(z^1)}{dz^1} &= \lambda \\ \delta E_{\theta,\theta'}\{\frac{dU(x^2(f(w^1,\theta)+y^2(\theta')))}{dx^2}\frac{dx^2(f(w^1,\theta)+y^2(\theta'))}{df(w^1,\theta)}\frac{df(w^1,\theta)}{dw^1}\} &= \lambda \\ f^1(w^1,\theta) + y^1 - x^1 - z^1 &= w^2 \end{aligned}$$

When the differentiability condition fails something similar holds with appropriately defined left-hand and right-hand derivatives.

These conditions can be rewritten in the more compact form:

$$\begin{aligned} \frac{dU(x^1)}{dx^1} &= \frac{dV(z^1)}{dz^1} \\ \delta E_{\theta,\theta'}\{\frac{dU(x^2(c^2))}{dx^2}\frac{df^2(w^1,\theta)}{dw^1}\frac{dx^2(c^2)}{dc^2}\} &= \frac{dU(x^1)}{dx^1} \end{aligned} \tag{5}$$

2.4 The Modified Euler Equation

The condition

$$\delta E_{\theta,\theta'}\{\frac{dU(x^2(c^2(\theta,\theta'))}{dx^2}\frac{df^2(w^1,\theta)}{dw^1}\frac{dx^2(c^2(\theta,\theta'))}{dc^2}\} = \frac{dU(x^1)}{dx^1} \tag{6}$$

where $c^2(\theta,\theta') = f(w^1,\theta) + y^2(\theta')$ ought to be reminiscent of the standard Euler equation in dynamic consumer maximization problems. Indeed, the only difference comes from the presence of the term $\frac{dx^2(c^2)}{dc^2}$: In our setting the standard Euler equation would take the form

$$\delta E_{\theta,\theta'}\{\frac{dU(x^2(c^2(\theta,\theta')))}{dx^2}\frac{df^2(w^1,\theta)}{dw^1}\} = \frac{dU(x^1)}{dx^1}.$$

The difference comes from the fact that there is some "dropped utility"—only part of the total expenditure on period 2 goods is valued by the period 1 self. Since $\frac{dU(x^2(f(w^1,\theta)))}{dx^2}$ and $\frac{df(w^1,\theta)}{dw^1}$ are always non-negative and $\frac{dx^2(c^2)}{dc^2} \leq 1$, his has the immediate implication that an observer who uses the modified Euler equation to estimate the decision-maker's discount factor as if it was the standard Euler equation (i.e proxying it by the ratio $\widehat{\delta} = \frac{E_\theta\{\frac{dU(x^2(c^2(\theta,\theta')))}{dx^2}\frac{df(w^1,\theta)}{dw^1}\}}{\frac{dU(x^1)}{dx^1}}$), would think that the person is more impatient than he actually is ($\widehat{\delta} \leq \delta$). Moreover it does not matter whether he uses an x or a z good to estimate the discount factor since, from the within period maximization $\frac{dU(x^2(c^2(\theta,\theta')))}{dx^2} = \frac{dV(z^2(f(w^1,\theta)))}{dz^2}$ and $\frac{dU(x^1)}{dx^1} = \frac{dV(x^2)}{dx^2}$

2.5 A revealing special case

Assume that U and V are both CRRA with the same coefficients:

$$\begin{aligned} U(x) &= \frac{x^{1-\alpha}}{1-\alpha} \text{ and} \\ V(z) &= A\frac{z^{1-\alpha}}{1-\alpha}. \end{aligned}$$

We already observed in this case $\frac{dx^2(c^2)}{dc^2}$ is a constant and the preferences (in the T-period case) have exactly the hyperbolic discounting form. Assume that we are in this case and therefore $x(c) = \frac{1}{1+q}c$, and $z(c) = \frac{q}{1+q}c$ where $q = A^{\frac{1}{\alpha}}$. Moreover let the person have no prospect for production ($F(k^1, \theta) = 0$) and no second period income. His only way to have some income in the second period is to save his income from the first period at the given interest rate R. Therefore (using the fact that the person consumes everything he has in period 2 and therefore $c^2 = Rw^1$), the above first order conditions can be rewritten in the form

$$\begin{aligned} \delta R \frac{1}{1+q}\left(\frac{Rw^1}{1+q}\right)^{-\alpha} &= (x^1)^{-\alpha} \\ y^1 - x^1(1+q) &= w^1 \end{aligned} \tag{7}$$

What is the effect of being more tempted on savings? In part that will depend on how we measure temptation. In the current CRRA model $\frac{q}{1+q}$ would appear to be a natural measure, since it represents the share of second period consumption that is wasted from period 1's point of view. The problem however is that changing q also changes the within period utility function in both periods and this might have implications for savings even in the absence of self-control problem. Conveniently, It turns out that in the CRRA case, a change in q shifts the period by period indirect utility functions in the same proportion (by a factor $(1+q)^{\alpha}$) in all periods and therefore has no effect on any of the intertemporal choices through this route. Therefore in this case we can focus on the effect of changing q.

Notice that $1+q$ enters 7 in 3 separate places: twice in the term $\delta R \frac{1}{1+q}(\frac{Rw^1}{1+q})^{-\alpha}$ and once through the expression $x^1(1+q)$. The effect through $\delta R \frac{1}{1+q}(\frac{Rw^1}{1+q})^{-\alpha}$ is the effect of an increase in temptation in the second period: Here there are two pieces—the $\frac{1}{1+q}$ term captures the substitution effect of the"tax" on every dollar that is spent in the second period (the part that goes in to z consumption). The $(\frac{Rw^1}{1+q})^{-\alpha}$ term captures the income effect of the "taxˊ"—you are consuming less x because there is more going to z. The reader will notice that these two effects have an exact parallel in the effects of a tax on interest earnings. Whether the net effect to discourage savings or to encourage them, depends on whether or not $\alpha \leq 1$. When it is, savings will go down with more temptation as a result of this effect. In other words, simply increasing the second period level of temptation keeping the first period level of temptation fixed might actually encourage savings through the "income" effect.

There is however a third effect: This is the effect through the $x^1(1+q)$ term, which captures the fact that a higher q in the first period means that keeping c^1 fixed, first period spending on x^1 has to go down. This is another "income" effect and clearly pushes for a higher level of first period spending (and hence less savings) keeping the second period level of q unchanged.

When q changes in both periods the income effects exactly cancel out in this CRRA case. This is easily seen by rewriting the first equation 7 in the form $\delta(\frac{R}{1+q})(\frac{c^2}{1+q})^{-\alpha} = (\frac{c^1}{1+q})^{-\alpha}$, and then noting that $(1+q)^{-\alpha}$ terms cancel from both sides to leave us with

$$\begin{aligned}\delta(\frac{R}{1+q})(c^2)^{-\alpha} &= (c^1)^{-\alpha} \\ y^1 - x^1(1+q) &= w^1\end{aligned}$$

We are now left with only one effect of raising q which is the substitution effect of the tax and this always leads to reduced savings: The net effect of increased temptation in this CRRA model is therefore exactly parallel to the effect of greater impatience (i.e lower δ).

The simplicity of this result owes a lot to the CRRA formulation. In the more general case the equivalent of raising q will be to move the entire $z(c)$ function up and it is not clear whether it is possible to do so without introducing shifts in the within period utility functions that would change savings behavior even there were no dropped utility.

For this reason, in the rest of this paper we will avoid making this kind of comparison: we will limit ourselives to showing that the presence of dropped utility introduces a set of possibilities that could not arise in its absence.

3 A key assumption about temptations

The logic so far highlights the effect of a temptation tax imposed by future selves. Though frameworks may differ, this logic is common to most models of self-control: a dissonance between how time t self would like $t+1$ to spend resources and how the $t+1$ self actually spends money. As we say in the introduction, we use this specific framework in order to be able to intuitively model how this dissonance—the temptation "tax"—changes as income rises. In the Modified Euler Equation (6), the tax is embodied by $\frac{dz^2(c^2)}{dc^2} = 1 - \frac{dx^2(c^2)}{dc^2}$: for every dollar spent $\frac{dz^2(c^2)}{dc^2}$ is the tax imposed by the period 2 self. Whether the tax declines with overall consumption therefore depends on whether $\frac{dz^2(c^2)}{dc^2}$ decreases or increases with income. Specifically if $\frac{dz^2(c^2)}{dc^2}$ decreases with total consumption, that is to say if $z^2(c^2)$ is concave, the impact of the tax decreases as consumption rises. Put differently, this offers an intuitive way to model the idea that as overall income rises, self control problems lessen. In the rest of the paper we will contrast this case which we will call the declining temptation case (DTC) with the alternative case where temptation does not decline ($\frac{dz^2(c^2)}{dc^2}$ is constant or increasing with c^2) which we will call the non-declining temptation case (NDTC). Note that NDTC includes the case where $z^2(c^2) = 0$, i.e there are no temptations. Obviously there are many other cases where z^2 is neither convex nor concave everywhere which fall into neither of these categories.

What does it mean for $z^2(c^2)$ to be concave? The following result offers an alternative and perhaps more intuitive characterization:

Proposition 1 *Assuming that V and U are three times differentiable everywhere and $z^2(c^2)$ is twice differentiable everywhere. Then $z^2(c^2)$ is strictly*

(weakly) concave everywhere if and only if $\widetilde{V}(x^2) = V(z^2(x^2))$ *is a strict (weak) concave transform of* $U(x^2)$.
Proof. *In appendix.* ■

In other words our condition is equivalent to assuming that V is more concave than U in this specific sense. So, for example, if both of them are CRRA, then we are asking the coefficient of relative "risk-aversion" on V to be greater than that on U.

Why should we believe that $z^2(c^2)$ is concave or equivalently V is more concave than U? One argument is based on the idea that most temptations are essentially visceral, reflecting desires that are rooted in our physiology (things like the the desire for sex, the craving for sweets and the love of fatty foods), and for that reason, relatively insensitive to the variety and range of quality that modern market economies offer (this should be true, for example, if the relevant physiological structures evolved in a world where the set of consumptiojussn choices were quite limited), As a result it is hard to spend a lot of money on temptation goods without hitting satiation. Another argument is based on the idea that most really expensive goods, like a sprts car or a house, are not really available for an impulse purchase in the same sense in which a cup of sugary tea or a trinket is—there are always multiple options that need to be examined and weighed, and all of that ensures that there is time for reflection and reconsideration.

In any case, whether this assumption holds is an question. We approach this in two ways. First, in the next Section we present a set of direct and testable consequences of $z^2(c^2)$ being concave. Testing them is a way to jointly test our model and this assumption. A second approach, which we take up in Section 4.8 is more direct. We describe how, using household consumption data and a set of choice experiments, one could plausibly determine the set of goods that are relatively more tempting. Examining whether these goods are a decreasing share of the budgets of the rich allows us to directly test the assumption that $z^2(c^2)$ is concave.

Finally, while we assume DTC implies that $z^2(c^2)$ is concave everywhere, it is not obvious that this is the most realistic assumption. For the ultra-poor very basic needs—simple calories—plausibly vdominate any temptation goods. Hence for this population $z^2(c^2)/c^2$ could be very low. A better assumption therefore is that $z^2(c^2)$ is S-shaped with a large cancave section, which is where the effects we emphasize become relevant. The assumption that $z^2(c)$ is concave everywhere is made to simplify the exposition.

3.1 A useful observation

In constructing examples it is sometimes useful to be able to pick a particular parametric form for the $z^2(c^2)$ function keeping the U function unchanged (i.e by just varying the V function). The following result shows tthat this can be done for a wide range of $z^2(c^2)$ functions.

Proposition 2 *Assume that the U function is known and fixed. Let $z^2(c^2)$ and $x^2(c^2)$ be a pair of non-negative valued strictly increasing functions defined on $c^2 \in [0, C]$ for some $C > 0$, such that $z^2(c^2) + x^2(c^2) = c^2$. Then there exists an increasing, differentiable and strictly concave function V defined on $[0, z^2(C)]$ such that the assumed $z^2(c^2)$ and $x^2(c^2)$ functions are the result of maximizing $U(x^2) + V(z^2)$ subject to a budget constraint $x^2 + z^2 = c^2$, and the conditions $(x^2 \geq 0, z^2 \geq 0)$*

Proof. *Define the function $g(z^2) = x^2(h(z^2))$ where the function $h(z^2)$ is the inverse of the function $z^2(c^2)$,which exists because of the strict monotonicity of z^2. Then define*

$$V(z^2) = \int_0^{z^2} U'(g(y))dy$$

Clearly $V'(z^2) = U'(g(z^2)) > 0$. And it is concave because when z^2 goes up $g(z^2)$ goes up and $U'(g(z^2))$ goes down. ■

Note that we did not require that $z^2(c^2)$ be differentiable, and for that reason, V may not be twice differentiable.

4 Implications of the model

4.1 Attributions of Impatience

A simple observation trivially follows from our key assumption. Recall that in subsection 2.4 we had defined $\widehat{\delta}$ to be the discount factor that an observer would (mistakenly) attribute to our decision-maker because he is basing his calculation on a model where there is no dropped utility. In the case where there is no uncertainty,

$$\widehat{\delta} = \delta[1 - \frac{dz^2}{dc^2}].$$

Since z^2 is assumed to be concave as a function of c^2 this tells us that those who are richer in the sense of consuming more in the second period will appear to be more patient to the observer despite the fact that everyone has the same δ. As will be shown later, second period consumption is monotonic in first period total income, and hence this could also be stated in terms of first period income (and also in terms of second period income).[9]

This framework, therefore, suggests an intuitive re-interpretation of the common observation that the poor seem to be more myopic than the non-poor that goes back at least to Irving Fisher (1932). This is a direct consequence of declining temptations. As we see in the section below, there is a secondary follow-on consequence. When the poor give into their temptations, it simply has larger consequences: In this framework, z(c) concave means, that it takes a smaller fraction of resources to satisfy one's temptations at high level

[9] We put it in terms of second period consumption because, as will emerge, first period consumption is not necessarily monotonic in first period wealth

of consumption than it does to satisfy one's temptations at a low level of consumption.\footnote{Note that though we do not model it here, one could include a self-control technology here. In that language, we would say that the poor require { \em greater} self-control since they would need to resist giving into the same temptations more than the rich.} All this goes to say, that two identically endowed people would appear myopic if born poor and less myopic if born rich, simply because the same failures (giving in to temptations) have greater consequences when poor. This last statement, of course, is a combination of the proposition above and a (yet to be proved) result of how wealth distributions evolve in the presence of declining temptations. We make this statement precise in Section 4.3 below.

4.2 Consumption smoothing

One of the most robust predictions of the standard model of savings is that an increase in future earnings (y^2) that leaves the return on investment unaffected will reduce today's savings and increase today's consumption (c^1). This is the direct result of the desire for consumption smoothing, induced by diminishing marginal utility. An increase in future income generates a desire to spend more both today and tomorrow. In our model, however, this need not be the case because there is a natural countervailing force. Notice that as y^2 rises, $\frac{dx^2}{dc^2}$ goes up and as a result, the right-hand-side of the Modified Euler equation could potentially even go up: The increased spending by the future self on x^2 increases today's self's desire to transfer income to the future and may even outweigh the effect of diminishing marginal utility.

Proposition 3 *Assume that second period income,y^2, is deterministic. Under NDTC, consumption today is increasing in future income: $\frac{dc^1}{dy^2} > 0$. Under DTC this need not be the case i.e. we might observe$\frac{dc^1}{dy^2} < 0$ over some range of y^2. Moreover we will only observe this pattern for people for whom y^1 and y^2 are sufficiently small.*

Proof. *Take the Modified Euler equation for this case*

$$\delta E_\theta\{\frac{df^2(w^1,\theta)}{dw^1}\frac{dU(x^2(c^2(\theta)))}{dx^2}\frac{dx^2(c^2(\theta))}{dc^2}\} = U'(x^1)$$

where $c^2(\theta) = f^2(y^1-x^1-z^1,\theta)+y^2$. In the NDTC, for any value of θ an increase in y^2 keeping $c^1 = x^1 + z^1$ fixed, increases $x^2 = c^2(\theta)$ and therefore depresses $U'(x^2(c^2(\theta)))$ for every realization of θ. Moreover it either depresses $\frac{dx^2(c^2(\theta))}{dc^2}$ or leaves it unchanged, for every realization of θ. Since $\frac{df^2(w^1,\theta)}{dw^1}$ is unchanged for each realization of θ, the left hand side is now less than the right hand side. Therefore x^1 has to go up to restore equality, and since $U'(x^1) = V'(z^1)$, z^1 must follow suit. Therefore a higher y^2 must be associated with a higher c^1.

In the DTC, i.e where $\frac{dx^2(c^2(\theta))}{dc^2}$ is increasing, the basic logic is very similar except that it is no longer obvious that the right hand side goes down. To see *this, take this case where $U(x) = \log x$. In that case, the product*

$$\frac{dU(x^2(c^2(\theta)))}{dx^2}\frac{dx^2(c^2(\theta))}{dc^2} = \frac{1}{x^2(c^2(\theta))}\frac{dx^2(c^2(\theta))}{dc^2}.$$

Whether x^1 goes up or goes down turns on whether $\frac{1}{x^2(c^2)}\frac{dx^2(c^2)}{dc^2}$ is increasing or decreasing as a function of c^2. A sufficient condition for this is that $x^2(c^2)$ is log-convex for $c^2 \leq \max_\theta f(y^1, \theta) + \overline{y}^2$, where $\overline{y}^2$ is the ceiling of the relevant range of y^2. It is easy to check that there are log-convex functions which are non-negative valued and satisfy $x^2(c^2) \leq c^2$ on any given finite range of c^2. Therefore from proposition 2 we can find a $V(z)$ function which makes $x^2(c^2)$ is log-convex. In such cases c^1 wil be decreasing in y^2.

Finally it is clear from the argument above that in order for c^1 to be decreasing in y^2, $x^2(c^2)$ has to be sufficiently convex to outweigh the natural concavity of the utility function. However since $x^2(c^2)$ is bounded above by c^2, there is a limit to how convex $x^2(c^2)$ can be on a unbounded domain—for large enough values of c^2, $x^2(c^2)$ must be approximately linear. Therefore c^1 can only be decreasing in y^2 for sufficiently low values of y^1 and y^2 (since c^2 is increasing in y^1 and y^2). ∎

This is a striking conclusion. It tells us that those who are sufficiently poor might actually react to the prospect of future income growth by beginning to save more. Conversely savings may actually be lower in exactly those times when cash will be needed the most in the future: faced with falling future incomes, people may boost consumption. This offers a possible interpretation of the idea that aspirations matter.

This result can be understood from a different angle, one that offers an intuition that helps us understand other results below. Consider an individual with a time-consistent utility function $u(c^1) + \delta u(c^2)$. Suppose, however, that instead of a investment technology that earns $f^2(w^1)$ he has only access to a technology that pays off $f^2(w^1)x'(f^2(w^1) + y^2)]$. Consider now the impact of an increase in y^2. There is the usual consumption smoothing motive that encourages an increase in c^1. But, here, however, there's an additional motive: the investment technology becomes more attractive: If the latter motive is strong enough, we might see the opposite of consumption smoothing.

4.3 Poverty Traps

This link between future income and savings has implications for first period income as well. First period income determines how much can be left behind for future selves. But since the savings invested for future consumption at time 1 (w^1) and income at time 2 (y^2) have similar effects, it is clear that a rise in y^1 could, in principle, have the same effect as an increase in y^2. If individuals start out with more income/wealth (which in this model are the same thing), they will be able to leave more to future selves. But this potentially creates a feedback effect: more wealth for time 2 means that x' is higher in that period, which creates an even greater desire to leave wealth to time 2. This feedback effect can be the source of a poverty trap:

Proposition 4 *Assume that there are no shocks i.e. that both θ_1 and θ_2 are constants. Then as long as we are in NTDC c^2 will be a continuous function of y^1. On the other hand in DTC, there may exist a $\overline{y}^1$, $0 < \overline{y}^1 < \infty$ such that c^2 jumps discontinuously upward at $\overline{y}^1$.*

Before we come to the main result, it is useful to observe that c^2 is always monotonic increasing with respect to y^1 and therefore the fact that c^2 jumps upwards if it jumps is automatic.

Lemma 5 *c^2 is monotonic increasing as a function of y^1.*

Proof. (of Lemma 5) To see this, suppose to the contrary, there exists y_0^1 and y_1^1 such that $c^2(y_0^1) > c^2(y_1^1)$ but $y_0^1 < y_1^1$. Let the values c^1 corresponding to this strategy be $c^1(y_0^1)$ and $c^1(y_1^1)$. Clearly $c^1(y_0^1) < c^1(y_1^1)$. Now consider an alternative consumption strategy for the person at $y_0^1$where he consumes $c^2(y_1^1)$ in the second period and sets $\widetilde{c}_0^1 = c^1(y_0^1) + \frac{c^2(y_0^1) - c^2(y_1^1)}{R}$. This must be dominated by what he actually chooses which implies that

$$\begin{aligned} & W(c^1(y_0^1)) - W(\widetilde{c}_0^1) \\ \geq \quad & \delta U(x^2(c^2(y_1^1))) - \delta U(x^2(c^2(y_0^1))) \end{aligned}$$

On the other hand the person at y_1^1 clearly prefers the pair $\{c^1(y_1^1), c^2(y_1^1)\}$ to the alternative of consuming $\widetilde{c}_1^1 = c^1(y_1^1) + \frac{c^2(y_0^1) - c^2(y_1^1)}{R}$ in the first period and $c^2(y_1^1)$ in the second. Therefore

$$\begin{aligned} & W(\widetilde{c}_1^1) - W(c^1(y_1^1)) \\ \leq \quad & \delta U(x^2(c^2(y_1^1))) - \delta U(x^2(c^2(y_0^1))) \\ \leq \quad & W(c^1(y_0^1)) - W(\widetilde{c}_0^1). \end{aligned}$$

However since $c^1(y_0^1)) - \widetilde{c}_0^1 = \widetilde{c}_1^1) - c^1(y_1^1)$, this contradicts the strict concavity of W. ■

Proof. (of Proposition 4) Consider the maximization problem:

$$U(x^1) + V(z^1) + \delta U(x^2(f^1(y^1 - x^1 - z^1) + y^2))$$

subject to

$$x^1 \geq 0, z^1 \geq 0.$$

As long as we are in NDTC, so that $x^2(c^2)$ is weakly concave, the strict concavity of $U(\cdot)$ and the weak concavity of $f(\cdot)$ (and the fact that these are all strictly increasing functions) guarantees that $U(x^2(f^1(y^1 - x^1 - z^1) + y^2))$ is a strictly convex (and decreasing) function of x^1 and z^1. $U(x^1)$ and $V(z^1)$ are also strictly concave. These conditions together guarantee that we have a strictly convex maximization problem, which tells us that the maximizers, x^1 and z^1, are always unique and vary continuously as a function of the parameters of the problem, y^1 and y^2. Hence the result.

In DTC, on the other hand this may not be true. To simplify the construction assume that $f^1(y^1 - x^1 - z^1) = R(y^1 - x^1 - z^1)$,with $\delta R > 1$. Also set $y^2 = 0$.

The way we will analyze this problem is by looking at how $c^2 - c^1$ behaves as a function of y^1. Clearly if c^2 is a continuous function of y^1, so is c^1 and $c^2 - c^1$.

Choose an $x^2(c^2)$ function which is convex (corresponding to the fact that we are in DTC) and such that there exist a c^*, $\infty > c^* > 0$, with $\delta R \frac{dx^2(c^*)}{dc^2} = 1$. Because $\delta R \frac{dx^2(c^*)}{dc^2} = 1$, if $c^2(y^1) = c^*$, then $c^1(y^1) = c^2(y^1) = c^*$. However for this to be true, $x^1 = x^2(c^*)$, must satisfy the second order condition

$$U''(x^1) + \delta R^2 U''(x^2(c^*))[\frac{dx^2(c^*)}{dc^2}]^2 + \delta R U'(x^1) \frac{d^2x^2(c^*)}{d(c^2)^2} \leq 0$$

Since $x^1 = x^2(c^*)$ and $\delta R \frac{dx^2(c^*)}{dc^2} = 1$, this expression can be rewritten as

$$\delta \frac{U'(x^2(c^*))}{x^2(c^*)}[\{x^2(c^*)\frac{U''(x^2(c^*))}{U'(x^2(c^*))}\}(\delta + 1) + \delta \frac{x^2(c^*)}{c^*}\{c^* \frac{\frac{d^2x^2(c^*)}{d(c^2)^2}}{\frac{dx^2(c^*)}{dc^2}}\}] \leq 0$$

$-x^2(c^*)\frac{U''(x^2(c^*))}{U'(x^2(c^*))}$ is the coefficient of relative risk aversion of the U function and measures its degree of concavity. $c^* \frac{\frac{d^2x^2(c^*)}{d(c^2)^2}}{\frac{dx^2(c^*)}{dc^2}}$ is a similar measure of convexity for $x^2(c^*)$. This condition therefore requires that the x function is not so convex as to overwhelm the concavity of U.

It is also clear that we can choose the $x^2(c^2)$ function that it satisfies the conditions of proposition 2 but violates the second order condition above at $c^2 = c^*$. Which means that the maximization problem has a local minimum at c^*: c^* cannot be the value of $c^2(y^1)$ for any y^1.

Therefore since $c^2(y^1)$ is an increasing function either c^2 it is discontinuous or $c^2(y^1) \to \bar{c} < c^*$ as $y^1 \to \infty$. But the latter case is impossible as long as we assume that

$$\frac{dx^2(c^2)}{dc^2} > \varepsilon > 0 \text{ for } c^2 \leq \bar{c}$$

and

$$U'(x) \to 0 \text{ as } x \to \infty.$$

This follows from the fact that if c^2 remains bounded above by $\bar{c}$, $x^1 \to \infty$ as $y^1 \to \infty$, and therefore the right hand side of the modified Euler equation goes to 0 (because of our assumption that $U'(x) \to 0$ as $x \to \infty$) while the left hand side remains bounded away from zero.

It follows that under these conditions, $c^2(y^1)$ must have a discontinuity and "jump over" c^*.

This result reinforces the discussion earlier about the myopia of the poor. Notice that here two individuals with { \em identical} discount rates but with different initial wealth levels can end up with very different levels of apparent patience: here the initially poor one will appear to be impatient and the initially rich one will appear to be patient. ■

4.4 Precautionary savings

With standard preferences it is well-known that an increase in income uncertainty in the second period (i.e a mean preserving spread in the distribution of y^2) will increase saings in a safe asset as long the single-period indirect utility function exhibits prudence. In our envronment the single-period indirect utility function would be given by

$$W(c) = \max_x U(x) + V(c - x).$$

If we were to assume that there is safe techonology for transfering wealth across time ($f^2(w^1, \theta) = Rw^1$, where R is a constant), a sufficient condition for there to be precautionary savings is that both U and V have non-negative third derivatives (and at least one of them is strictly positive).

With our kind of preferences this condition is no longer sufficient. To see this recall the modified Euler equation for this case:

$$\delta R E_{\theta'} \{ \frac{dU(x^2(c^2(\theta')))}{dx^2} \frac{dx^2(c^2(\theta'))}{dc^2} \} = U'(x^1).$$

In other words while we do have the standard precautionary savings effect coming in as long as $\frac{dU(x^2(c^2))}{dx^2}$ is convex as a function of c^2, there is two additional potentially countervailing effects. One comes from the fact that $\frac{dx^2(c^2)}{dc^2}$ may not be convex as a function of c^2. The other comes from the fact that in the DTC $\frac{dU(x^2(c^2))}{dx^2}$ and $\frac{dx^2(c^2)}{dc^2}$ move in opposite directions when c^2 goes up. This last effect has an very intuitive explanation: The whole point of precautionary savings is to raise x consumption levels in the state of the world when c^2 is particularly low. But in the DTC $\frac{dx^2(c^2)}{dc^2}$ is particularly low when c^2 is low and as result saving more does not help very much in terms of raising x consumption in when c^2 is low. Therefore the DTC partially defeats the purpose of saving more to protect against low c^2 states. The following example this clear

4.4.1 An Example

Let $U(x) = \ln x$ and $x^2(c^2)$ be described

$$\begin{aligned} x^2(c^2) &= \alpha c^2, c^2 \leq \underline{c}, 0 < \alpha < 1 \\ x^2(c^2) &= Ae^{\beta c^2}, \overline{c} > c^2 > \underline{c} \\ x^2(c^2) &= \widetilde{\alpha} c^2 - \gamma, c \geq \overline{c}, 0 < \widetilde{\alpha} < 1, \gamma > 0 \end{aligned}$$

To ensure $x^2(c^2)$ is differentiable, continuous and increasing everywhere, assume that

$$\begin{aligned}\alpha\underline{c} &= Ae^{\beta\underline{c}} \\ \alpha &= \beta Ae^{\beta\underline{c}} \\ \widetilde{\alpha}\overline{c} &= Ae^{\beta\overline{c}} - \gamma \\ \widetilde{\alpha} &= \beta Ae^{\beta\overline{c}}\end{aligned}$$

which together imply that $\overline{c}$ has to be equal to $\frac{1}{\beta}$ and that $\alpha < \widetilde{\alpha}$. Under these assumptions

$$\frac{dx^2(c^2)}{dc^2} = \beta x^2 \text{ for } \underline{c} < c^2 < \overline{c}.$$

Therefore $\frac{dx^2(c^2)}{dc^2}$ is increasing as long as c^2 is between $\underline{c} < c^2 < \overline{c}$, and constant otherwise. We assume that the initial (i.e. before the mean preserving spread in the distribution of y^2 and hence c^2) distribution of y^2 is such that $\underline{c} < c^2 < \overline{c}$. for all realizations of θ'. Then this will continue to be true for a small perturbation in the distribution of y^2 and we can assume that $\frac{dx^2(c^2)}{dc^2} = \beta x^2$ (and therefore DTC) everywhere in the relevant range.

Substituting this in the modified Euler equation gives us

$$\begin{aligned}\delta RE_{\theta'}\{\frac{1}{x^2(c^2(\theta'))}\beta x^2(c^2(\theta'))\} &= U'(x^1) \\ &\quad \textit{or} \\ \beta\delta R &= U'(x^1)\end{aligned}$$

In other words, the right hand side is now a constant: Shifts in the distributionof y^1 (mean preserving or otherwise) have no effect on the decision to save as long as c^2 remains above $\overline{c}$.

What would we have found under conventional preferences? $U(x) = \ln x$, so $U''' > 0$. $V(z)$ has to defined to generate the chosen $x^2(c^2)$ function:

$$V(z^2) = \int_0^{z^2} U'(g(y))dy$$

where $x^2 = g(z^2)$ is the relationship between the optimizing values of x^2 and z^2 for the same value of c^2. Therefore

$$\begin{aligned}V'(z^2) &= U'(g(z^2)) \\ V''(z^2) &= U''(g(z^2))g'(z^2) \\ V'''(z^2) &= U'''(g(z^2))[g'(z^2)]^2 + U''(g(z^2))g''(z^2)\end{aligned}$$

Now since $\frac{dx^2}{dc^2} = \beta x^2$ and $\frac{dz^2}{dc^2} = 1 - \beta x^2$,

$$\begin{aligned}g'(z^2) &= \frac{dx^2}{dz^2} = \frac{\beta x^2}{1-\beta x^2} \\ &\quad \textit{and} \\ g''(z^2) &= \frac{dx^2}{dz^2} = \frac{\beta}{(1-\beta x^2)^2}\frac{\beta x^2}{1-\beta x^2}\end{aligned}$$

Therefore

$$V'''(z^2) = \frac{2}{(x^2)^3}(\frac{\beta x^2}{1-\beta x^2})^2 - \frac{1}{(x^2)^2}\frac{\beta}{(1-\beta x^2)^2}\frac{\beta x^2}{1-\beta x^2}$$
$$= \frac{\beta^2}{x^2(1-\beta x^2)^2}[2 - \frac{1}{1-\beta x^2}]$$

This is positive in the relevant range as long as

$$\frac{1}{1-\beta x^2(\bar{c})} < 2.$$

Moreover for c^2 outside the range $[\underline{c}, \bar{c}]$ z is linear in x, and therefore V''' has the same sign as U'''. Therefore the condition for precautionary savings with conventional preferences (that both U''' and V''' are strictly positive) holds everywhere. Yet there is no precuationary savings.

4.5 Implications for the Structure of Investments

Standard utility theories have some useful implications about investment demand, clarifying which features of an investment matter and how they might be traded off against other. For example, they highlight the importance of risk diversification and the minimum scale of the investment (increasing returns). The addition of temptation highlights another feature of the investment opportunity: the *maximum* scale. Individuals facing declining temptations will be unwilling to invest in high returns investments if the scale is too small. This captures the lay intuition that investments may be unimportant unless they significantly change one's circumstances.

Assume that the period one self has been offered access to a set of new investment technologies (in addition to what was already available to him). We are interested in whether he would take it up. These new investments are described three features. Assume that each investment ι is described by its return $R(\iota)$, minimum size $s(\iota)$, and maximum size $S(\iota)$.[10] If he undertakes the investment ι, he can choose an investment level I between $s(\iota)$ and $S(\iota)$. Period 1 self will then have I units less to spend but period 2 self will receive $R(\iota)I$ returns. In this highly abstract setup, we can examine the rank-ordering of investments. If individuals are willing to undertake i what ι' would they also be willing to undertake?

To analyze this rigorously we focus on a setting where both y^1 and y^2 are deterministic, borrowing is ruled out (allowing some borrowing would not change anything essential, but makes some of the arguments more tedious) and there is "base" investment technology, $f(w^1) = R_0 w^1, R_0 > 1$. The investor is now offered the option of investing in *one* additional technology with the understanding that he still has the option of investing as much as he wants in the base technology (subject, as before, to the constraint that he cannot borrow.

[10] The model also has implications for the timing of investment which we do not investigate in this current version.

Proposition 6 *In this setting under NDTC if the investor is willing to undertake an investment* $\iota = \{R(\iota), s(\iota), S(\iota)\}$ *then he will always be willing to undertake an investment* $\iota' = \{R(\iota'), s(\iota'), S(\iota')\}$ *as long as* $R(\iota') \geq R(\iota)$ *and* $s(\iota') \leq s(\iota)$. *In other words, minimum scale and returns summarize the investment. In contrast, if* $S(\iota) > S(\iota')$, *under DTC there exist situations where this is not true even if* $R(\iota') > R(\iota)$ *and* $s(\iota') < s(\iota)$.
Proof. *Define* $W(c^1, c^2)$ *to be the period 1 self's maximand, i.e.*

$$W^1(c^1, c^2) = W(c^1) + U(x^2(c^2)).$$

In the NDTC, since x^2 *is a concave function of* c^2, $W^1(c^1, c^2)$ *is a strictly concave function of the vector* (c^1, c^2). ■

Suppose period 1 self's optimal choice when offered the option ι *is to invest an amount* $I > 0$ *in* ι *and to consume an amount* c^1 *in period 1. The amount he invests in the base technology is therefore* $y^1 - I - c^1$. *Clearly in the absence of credit* $y^1 - I - c^1$ *must be non-negative.* c^2 *in this case is given by*

$$c^2 = IR(\iota) + (y^1 - I - c^1)R_0 + y^2$$

If instead he had chosen not to invest in ι, *he would have consumed an amount* $\widetilde{c}^1$ *in period 1 and invested* $y^1 - \widetilde{c}^1$ *in the base technology. Therefore*

$$\widetilde{c}^2 = (y^1 - \widetilde{c}^1)R_0 + y^2.$$

By the fact that the investor chose to invest

$$W(c^1, c^2) \geq W(\widetilde{c}^1, \widetilde{c}^2)$$

Now consider the vector $(\lambda c^1 + (1-\lambda)\widetilde{c}^1, \lambda c^2 + (1-\lambda)\widetilde{c}^2), 1 > \lambda > 0:$ *Clearly, by the strict concavity of W,* $W(\lambda c^1 + (1-\lambda)\widetilde{c}^1, \lambda c^2 + (1-\lambda)\widetilde{c}^2) > W(\widetilde{c}^1, \widetilde{c}^2)$. *The question is whether* $(\lambda c^1 + (1-\lambda)\widetilde{c}^1, \lambda c^2 + (1-\lambda)\widetilde{c}^2)$ *is in the option set. Note however that we can write*

$$\begin{aligned} & \lambda c^2 + (1-\lambda)\widetilde{c}^2 \\ = \; & \lambda(IR(\iota) + (y^1 - I - c^1)R(0) + y^2) \\ & +(1-\lambda)((y^1 - c^1)R(0) + y^2) \\ = \; & \lambda IR(\iota) + (y^1 - \lambda I - c^1)R(0) + y^2 \end{aligned}$$

In other words as long as the investment level λI *is in the option set (i.e,*$s(\iota) \leq \lambda I \leq S(\iota)$*)* $(\lambda c^1 + (1-\lambda)\widetilde{c}^1, \lambda c^2 + (1-\lambda)\widetilde{c}^2)$ *is feasible and dominates just investing in the base asset.*

Now consider an alternative asset ι' *with returns* $R(\iota') \geq R(\iota)$. *Clearly*

$$\begin{aligned} & W(\widetilde{c}^1, \widetilde{c}^2) \\ < \; & W(\lambda c^1 + (1-\lambda)\widetilde{c}^1, \lambda c^2 + (1-\lambda)\widetilde{c}^2) \\ \leq \; & W(\lambda c^1 + (1-\lambda)\widetilde{c}^1, \lambda IR(\iota') + (y^1 - \lambda I - c^1)R_0 + y^2) \end{aligned}$$

which is what you would get by investing the same amount (λI) *in asset* ι' *after choosing the same amount of first period consumption. Therefore as long as we can make sure that the amount invested in asset* ι' *is feasible, i.e set* λI *between* $s(\iota')$ *and* $S(\iota')$*, then there will be investment in* ι'*. But since* $s(\iota') \leq s(\iota)$*, it will always be possible to set* λ *so that* $s(\iota') \leq \lambda I \leq S(\iota')$*. Therefore there will always be some investment in* ι'*.*

Under DTC, $W(c^1, c^2)$ *is not necessarily concave. To see where things might break down when* $W(c^1, c^2)$ *is not concave consider the following special preferences:*

$$\begin{aligned} V(z) &= az, z \leq \overline{c}, \\ &= a\overline{c}, z > \overline{c} \\ & \quad \textit{and} \\ U(0) &= 0, 0 < U'(0) < a, U''(x) < 0 \end{aligned}$$

In the second period, these preferences imply that

$$\begin{aligned} z^2(c^2) &= c^2, z \leq \overline{c} \\ &= \overline{c}, z > \overline{c}. \end{aligned}$$

Given these preferences anyone with y^1 *and* y^2 *such that* $R_0 y^1 + y^2 < \overline{c}$*, will not save as long as the base technology is the only available technology, because he faces a* $z'(c)$ *of 1. Moreover, given that* $R_0 > 1, y^1$ *must be less than* $\overline{c}$ *and therefore period 1's self will consume all of* y^1 *in the form of the z good. His two period utility is therefore* ay^1

Next, assume that he is willing to invest an amount I *in technology* ι*. This requires that*

$$R(\iota)y^1 + y^2 > \overline{c}$$

that

$$\delta U(R(\iota)y^1 + y^2 - \overline{c}) > ay^1.$$

and that

$$\delta R(\iota)U'(R(\iota)y^1 + y^2 - \overline{c}) > a.$$

Clearly we can find a $R(\iota)$ *large enough for which these conditions hold.*

Finally assume that there is an ι' *such that* $R(\iota' > R(\iota)$*,* $s(\iota') < s(i)$ *and* $S(\iota) < S(\iota)$*. Now if*

$$R(\iota')S(\iota') + y^2 < \overline{c}$$

there will obviously not be any investment in ι'*,even though it has a higher per dollar return and lower minimum scale. The logic of this construction makes clear that it can easily be extended to the case where both* V *and* $z^2(x^2)$ *are differentiable functions.*

This result is important, we feel, for two reasons. First, it is noteworthy that some of the high return investments that have been noted in the literature are capped. Fertilizer, for example, may earn very high rates of return and may not be lumpy, but it's maximum scale is limited. While Duflo, Kremer and Robinson describe very high { \em returns}, the actual level of money earned is small, because the amount of fertilizer one can place is limited by one's land. Similarly, Kremer, Lee and Robinson () articulate that stocking behavior on phone cards illustrate unexploited high rates of return investments. But as with fertilizer, these are investments that have small maximum scale. The optimal stocking behavior necessitates some more cards, but holding more than a few more would not be optimal. Second, it is noteworthy that when combined with credit constraints, this model can imply unwillingness to invest even in investments that are of maximum scale. If an investment opportunity allows unlimited scale but credit constraints only allow investment up to a smaller scale, then these investments are effectively capped in scale. Even if one can physically they do not have a cap, the credit constraints create an effective maximum scale. These two observations combined provide some potential explanations for the high rates of return on capital that we have found. In short, these results suggest that in interpreting investment decisions, empirical work should also consider the maximum feasible scale of these investments.

4.6 The Role of Credit

Declining temptations also have implications for the benefits and costs of credit. In the traditional model, access to credit is clearly good: it increases the opportunity set. In models with self-control problems, credit can potentially hurt. Specifically, today's self could be made worse off if tomorrow's self has access to credit. This general feature of self-control models has more specific implications if we focus on declining temptations.

To fully capture commitment benefits in this context, we need to introduce a third ("zero") period. In the existing two period model, commitment has benefits only if the first period can restrict the *type* of consumption (x goods over z goods) in the second period. But in this section and the ones below, we will be interested in what are effectively cruder commitment mechanisms: the restriction of overall level of consumption. In two periods, this would be a meaningless concept since the second period self would consume all wealth in any case. In a three period model, however, the zero period self can undertake actions that restrict period one self's ability to consume.

Since we are interested only in the investment and commitment demands of this zero period self, we assume this self has no consumption. Instead he or she merely maximizes $U(x^1) + \delta U(x^2)$.[11] The other two periods are as before.

In this subsection, we focus on the case where the zero period has a single decision: to allow or disallow period one to have access to credit. Specifically, we assume that period 1 self has no access to credit markets but zero period self may give him specific types of access. We will then ask what types of access will zero period self be willing to allow. Suppose that a loan λ has three features:

[11] In what follows, what matters is that the period 0 self is effectively more patient than period 1 self, so technically the period 0 self could also maximize $U(x^1) + cV(z^1) + \delta[U(x^2) + cV(z^2)]$ for some $c > 0$ without changing our analysis in any signficant way.

$r(\lambda)$, the interest rate, a *maximum* loan size $L(\lambda)$, and a *minimum* loan size $l(\lambda)$. If period 1 self is allowed a loan of type λ, he can choose to borrow some amount between $l(\lambda)$ and $L(\lambda)$ and have period self repay r times that amount.

We will again ask question of the type: if the zero period self allows loan type λ, will he allow loan type λ'? The key insight is that the *shape* of temptation places structure on the demand for commitment. Constant temptation implies that zero period self will be interested in placing limits on maximum loan size $L(\lambda)$. But declining temptation implies that zero period self will have an additional interest in placing limits on minimum loan size $l(\lambda)$.

Proposition 7 *Under NDTC. If zero period self is willing to allow loan* $\lambda = \{r(\lambda), l(\lambda), L(\lambda)\}$*, he will always allow loan* $\lambda' = \{r(\lambda'), l(\lambda').L(\lambda')\}$ *as long as* $r(\lambda) = r(\lambda'), L(\lambda') \leq L(\lambda), l(\lambda') \leq l(\lambda)$. *Under DTC there will exist situations where he is willing to allow a loan* $\lambda = \{r(\lambda), l(\lambda), L(\lambda)\}$*, but not a loan* $\lambda' = \{r(\lambda'), l(\lambda').L(\lambda')\}$ *where* $r(\lambda) = r(\lambda'), l(\lambda) = l(\lambda')$ *but* $L(\lambda') < L(\lambda)$. *There will also exist situations where he is willing to allow* λ *but not* λ' *where* $r(\lambda) = r(\lambda'), L(\lambda') = L(\lambda), l(\lambda) > l(\lambda')$.

Proof. *NDTC: In this case since* $x^1(c^1)$ *and* $x^2(c^2)$ *are both concave, Period 0's utility function*

$$\Pi_0(L) = U(x^1(y^1 + L)) + \delta U(x^2(y^2 - Lr(\lambda))))$$

is concave as a function of L*, the actual loan amount. Assume that period 0 self permits a loan product* λ (r, l, L) *but not a loan product* λ' (r, l', L') *where* $l' \leq l$*, and* $L' \leq L$*. This means that*

$$\Pi_0(\widetilde{L}) = U(x^1(y^1 + \widetilde{L})) + \delta U(x^2(y^2 - r\widetilde{L})) \geq U(x^1(y^1)) + \delta U(x^2(y^2)) = \Pi_0(0)$$

but

$$\Pi_0(\widetilde{L}') = U(x^1(y^1 + \widetilde{L}')) + \delta U(x^2(y^2 - r\widetilde{L}')) < U(x^1(y^1)) + \delta U(x^2(y^2)) = \Pi_0(0)$$

where $\widetilde{L}, \widetilde{L}'$ *are the loan amount actually chosen under* λ, λ'. *Now we know that Period 1 self will want to choose the same loan amount under both* λ, λ'*, since the interest rate is the same and if does choose the same amount, Period 0 might as well have chosen* λ'. *The interesting case is where* $\widetilde{L} \neq \widetilde{L}'$*, and therefore either* $\widetilde{L} \geq L(\lambda') \geq \widetilde{L}'$ *or* $\widetilde{L} \geq l(\lambda) \geq \widetilde{L}'$. *In either case* $\widetilde{L}'$ *is between* $\widetilde{L}$ *and a loan size of 0. But then, by the concavity of* $\Pi_0(L)$, $\Pi_0(\widetilde{L}')$ *has to be bigger than* $\Pi_0(0)$*, which directly contradicts what we said above. This contradiction proves that* λ' *will be chosen if* λ *is chosen.*

DTC: To show that this is not necessarily true in DTC, consider the example from the prevous section where case where

$$\begin{aligned} V(z) &= az, z \leq \bar{c}, \\ &= a\bar{c}, z > \bar{c} \\ &\quad \textit{and} \\ U(0) &= 0, 0 < U'(0) < a, U''(x) < 0 \end{aligned}$$

In the second period, these preferences imply that

$$\begin{aligned} z^2(c^2) &= c^2, z \leq \bar{c} \\ &= \bar{c}, z > \bar{c}. \end{aligned}$$

Also $r(\lambda) = 1$. *Therefore*

$$x^1 + z^1 + x^2 + z^2 = y^1 + y^2$$

Assume that $y^1 = \alpha\bar{c}$ $(0 \leq \alpha \leq 1)$, $y^2 = \bar{c} + k$ *and there is no borrowing allowed. Call this scheme of* $(1, 0, 0) = \lambda_0$. *Then*

$$\begin{aligned} \Pi_0(\lambda_0) &= \delta U(k) \\ \Pi_1(\lambda_0) &= V(\alpha\bar{w}) + \delta U(k) \\ \Pi_2(\lambda_0) &= U(k) + V(\bar{w}) \end{aligned}$$

Now assume that there exists a loan $\lambda_1 = (1, l(\lambda_1), L(\lambda_1))$, *where* $L(\lambda_1) < (1-\alpha)\bar{w}$. *For simplicity call this* $\gamma\bar{w}$, $\gamma < 1 - \alpha$. *Then*

$$\begin{aligned} \Pi_0(\lambda_0) &= \delta U(k - \gamma\bar{w}) \\ \Pi_1(\lambda_0) &= V((\alpha+\gamma)\bar{w}) + \delta U(k - \alpha\bar{w}) \\ \Pi_2(\lambda_0) &= U(k - \gamma\bar{w}) + V(\bar{w}) \end{aligned}$$

Therefore, since the period 0 self does not stand to benefit (and actually is hurt) from maximal loan size until the threshold of $(1-\alpha)\bar{w}$, *a necessary condition for the loan he would allow would require* $L(\lambda) > (1-\alpha)\bar{w}$. *Assume* $L(\lambda_2) = (1-\alpha)\bar{w} + \varepsilon$. *Then*

$$\begin{aligned} \Pi_0(\lambda_0) &= U(\varepsilon) + \delta U(k - L(\lambda_2)) \\ \Pi_1(\lambda_0) &= U(\varepsilon) + V(\bar{w}) + \delta U(k - L(\lambda_2)) \\ \Pi_2(\lambda_0) &= U(k - L(\lambda_2)) + V(\bar{w}) \end{aligned}$$

Provided $U(\varepsilon) \geq \delta[U(k) - U(k - L(\lambda_2))]$, *which holds for* α *sufficiently close to 1 since* U *is strictly concave, we have shown that* $\exists\lambda_2 = (r(\lambda_2), l(\lambda_2), L(\lambda_2))$ *and* $\lambda_1 = (r(\lambda_1), l(\lambda_1), L(\lambda_1))$ *with* $r(\lambda_1) = r(\lambda_2)$, $l(\lambda_1) = l(\lambda_2)$, $L(\lambda_1) < L(\lambda_2)$ *where period 0 accepts* λ_2 *but not* λ_1. ■

The intuition behind this result is simple. When temptations are constant, the only fear for the zero-period self is over-borrowing. This has two consequences: (i) Exaggerating the difference in consumption between period one and two selves and (ii) Engaging in borrowing at a higher rate than zero period self would want.[12] Zero must weight these costs against the potential for borrowing to facilitate investments. As a result, zero will only want to permit borrowing up to the point where diminishing marginal product induces one

[12] zero discounts at δ while 1 discounts at $\delta(1 - z'(c))$.Interest rates between those would induce inefficient borrowing.

to borrow simply for consumption. When temptations are declining, however, there is an offsetting force. One may be unwilling to invest much at small amounts because the income effect and the consequent decline in temptation in period two is small. This is the observation from the previous sub-section. A big loan may induce investment because one now sees the investment as being big enough that it is "worthwhile". So zero may be willing to allow loans that allow more borrowing over ones that allow less. A related, more subtle, intuition arises for why minimum size matters. Minimum size restrictions work when they effectively guarantee that one is borrowing only to make big invest. One may have a preference ordering which first favors borrowing small amounts for consumption, then making a big investment and then not borrowing. In this case, zero can guarantee his preferred outcome (making a big investment) by ruling out the small consumption loans.

It is worth noting that a very similar effect arises even in the constant temptation model if there is a source of increasing returns, for example if there are lumpy investments or consumption durables which have high returns. In this case, zero again has an incentive to allow loans of size just big enough to permit investment in these lumpy investments.

These results, we feel, are extremely helpful in parsing out credit contracts as we see them. Consider two different contracts: microfinance and credit cards. Micro-finance loans are lumpy and have fixed minimum sizes. Individuals typically must borrow a large lump-sum all at once. Credit cards, on the other hand, allow for small amounts of borrowing every day. This model, with declining temptations (or constant temptations combined with lumpy investments), would differentiate the commitment benefits of these two. It illustrates how the minimum size restrictions of the micro-finance contract may actually solve temptation problems, while credit cards' licensing of borrowing in small sums may actually exaggerate temptation problems.[13]

4.7 Money lenders and debt traps

We illustrate how monopolistic money-lenders would be able to create debt-traps for the poor, preventing them from adopting high-return technologies. This is in contrast to traditional models, where debt traps are not feasible, as discussed by Srinivasan (). This produces a behavioral resolution to the dispute on the feudal economy (Bhaduri, Srinivasan).

Assume that there is a time inconsistent agent and a money lender from whom she is currently borrowing. An investment opportunity with gross return ρ arises in the village. The timing is as follows:

1. $t = 0$: The money lender chooses R_0 and R_1. Knowing this, the individual chooses whether or not to invest $I - i$.

[13] We do not explore here a different commitment feature of debt: the forced repayment that it implies. We conjecture that on this dimension as well, contracts which appear as micro-finance contracts (big lump sum, combined with small repayment installments) help solve temptation problems more so than credit card like contracts (small trickles of borrowing, combined with a big repayment.

2. $t=1$: Individual chooses whether or not to invest i and consumes x^1, z^1.

3. $t=2$: Individual receives ρI and repays $R(I-i)$ if an investment was made and consumes x^2, z^2.

Theorem 8 *Let $\mathcal{U}, \mathcal{V}$ be classes of (utility) functions inducing $z'(w)$ decreasing in the modified Euler Equation. Let $\mathcal{D}=(0,1)$, $\mathcal{Y}=\mathbb{R}_{>0}$, $\mathcal{I}=\mathbb{R}_{>0}$, $\varrho \in \mathbb{R}_{>0}$, and $\mathcal{R}=(0,\varrho]$. Define the environment space Θ as*

$$\Theta = \left\{U, V, \delta, y^1, y^2, I, \rho, R_0, R_1\right\}_{U\in\mathcal{U}, V\in\mathcal{V}, \delta\in\mathcal{D}, y^1, y^2\in\mathcal{Y}, I, i\in\mathcal{I}, \rho\in\varrho, R_0, R_1\in\mathcal{R}}$$

Assume the timing structure is as above, $y^1 > i$, let $\mathbf{1}_{\{R|\theta\}}$ be an indicator function of whether the individual takes up the investment opportunity.

Then $\exists\theta\in\Theta$ such that $R^*(\theta) \in \arg\sup_{R\in\mathcal{R}} \pi(R)$ has $\vartheta(R^*(\theta)|\theta) = 0$.

We now provide a simple example of the phenomenon. Assume that the individual has u, v such that $\frac{c^1}{c^2}$ is either one of two constants.[14] If the period 1 self is patient, then $\frac{c^1}{c^2} = k_p$, while if the period 1 self is impatient, then $\frac{c^1}{c^2} = k_i$, where $k_p < k_i$. One can think of k as $k = \frac{1}{\hat{\delta}} = \frac{1}{\delta\left[1-\frac{dz^2}{dw^2}\right]}$. If sufficient wealth is transferred along the optimal path to period 2, i.e. $w^2 > \bar{w}^2$, then period 1 self is patient.

$$k = \mathbf{1}_{\{w^2(R)\geq\bar{w}^2(R)\}} k_p + \left(1-\mathbf{1}_{\{w^2(R)\geq\bar{w}^2(R)\}}\right) k_i$$

Notation:

1. R^N denotes the optimal interest rate charged by the money lender in the counterfactual – had the investment opportunity not arrived to the village. One can think about this as the interest rate the money lender was charging before the technology arrived.

2. $L^N = L^N\left(R^N\right)$ is the loan amount taken in the counterfactual (or before the technology arrived). We can show that this will be achievable even with the investment opportunity available.

[14] Such an equilibrium consumption pattern can be obtained by creating u, v through pasting together CRRA functions.

$$\begin{aligned} U(x) &= \frac{x^{1-\gamma}}{1-\gamma} \\ V(z) &= \mathbf{1}_{\{z\leq\tau\}} A\frac{z^{1-\gamma}}{1-\gamma} + \left(1-\mathbf{1}_{\{z>\tau\}}\right) a\frac{z^{1-\gamma}}{1-\gamma} \end{aligned}$$

with $A > 1 > a$. The equilibrium solution will have

$$z = \mathbf{1}_{\{z<\tau\}}\beta x + \left(1-\mathbf{1}_{\{z>\tau\}}\right)\theta x$$

where $\beta > \theta$.

3. $R_0, R_1, k_i, k_p, \bar{w}^2$ as above.

Assumptions:

1. At $R_0 = R_1 = R^N$ the individual borrows to make the investment.
2. $I < L^N$
3. $\rho \in (k_p, k_i)$. This rules out situations where the
4. ρ is sufficiently large such that $w^2\left(R^N, R^N\right) > \bar{w}^2\left(R^N\right)$
5. $R^N > k_p$

The money lender always has the opportunity to force the individual into a situation as if there was no investment opportunity. He can simply offer (R_0, R) such that $R_0 = \rho$. Because in the world without the investment opportunity R^N was assumed to be the profit maximizing rate, the money lender will then offer $\left(R_0, R^N\right)$. In this situation, the money lender receives $\left(R^N - 1\right) L^N$.

Meanwhile, if the money lender allows the agent to invest, he must make at least $R^N L^N$. However, in the world with investment, he makes $R_1 (I - i)$. Therefore for the money lender to encourage investment, he must have $R_1 \geq \frac{R^N L^N}{I-i}$. Observe that ρ is an upper bound on R_1. Therefore, the money lender can at most make $(\rho - 1)(I - i)$ in this scenario.[15] A debt trap occurs whenever $(\rho - 1)(I - i) < \left(R^N - 1\right) L^N$.

4.8 Empirical Test of the Model

The above propositions provide qualitative predictions. But the model also provides a few specific quantitative tests. Here we outline how those tests might appear.

First, we can actually test whether temptation is in fact declining. To do this, suppose there are several goods g_i. Suppose that each good has an xcomponent and a z component. Specifically suppose that one unit of good i provides x_i fraction of the non-temptation and z_i fraction of the temptation good. Moreover, suppose that what constitutes x and z goods is common across individuals.[16] Now consider the following experiment. Suppose that we offer to a set of individuals a choice between 1 unit of good i and d_i units of good i in

[15] Moreover, since this holds $\forall \rho \in (k_p, k_i)$, then the supremum of the money lender's profits by inducing investments (over the range of admissible returns to investment) is

$$(k_i - 1)(I - i)$$

which tells us that as k_i increases, the money lender can extract more rents and therefore for a given $\left(R^N, L^N\right)$, very high return investments are more likely to have $(k_i - 1)(I - i) > \left(R^N - 1\right) L^N$. Therefore, money lenders would support exceedingly high return investments, while they might discourage moderate return investments.

[16] A weaker assumption that would still work is to assume there is a common component with individuals varying in an iid way around this common component.

one period. Assume, as is common in all discount rate experiments, that there is non-fungibility across time and goods so that individuals view this offer as a genuine increase of either 1 unit today or $d_i$units tomorrow.[17] Define $\overline{d}_i$ to be the average d_i that makes individuals indifferent to this tradeoff, i.e. half the individuals choose 1 unit today and half choose $\overline{d}_i$ units in one period. These experiments allow us to array goods according to how much temptation they provide. Goods that provide more temptation should show larger $\overline{d}_i$: one would need a large quantity in the future to induce one to give up one unit today. The key assumption of our model can be empirical tested by looking at the Engel curve for each goods: declining temptation implies that the Engel curve should be steeper for goods with lower $\overline{d}_i$. Note that this is not a hard-wired assumption. The Engel curve captures how demand for a particular good varies with income, whereas $\overline{d}_i$ measures the discount rate associated with a particular good.

A second quantitative test of our model is based on our assertion that the apparent patience difference between poor and rich is due to the composition of consumption rather than genuine difference in patience. To test this, we would offer trade offs of 1unit of money today versus d_munits tomorrow and use this to back out an apparent discount rate for money δ_m. We would then offer (again, under the assumption of fungibility), 1 unit of an x good (identified as above) today versus d_0 units tomorrow. This allows us to back out an apparent discount rate for x goods δ_x. We then predict that $\frac{\delta_m}{\delta_x}$ is declining in income: the poor are much more impatient in money than in x goods and this gap closes as income increases.

5 Conclusion

To be added

[17]This assumption is common in all discount rate experiments that are undertaken for money, e.g. one dollar today or two dollars in one month. Such experiments are meaningless if money were fungible in time for example.

6 Appendix

Proof of Proposition 1

Proof. Since $U'(x^2(c^2)) = V'(z^2(c^2))$, and $x^2(c^2)+z^2(c^2) = c^2$, $\frac{dz^2}{dc^2} = \frac{U''(x^2(c^2))}{V''(z^2(c^2))+U''(x^2(c^2))}$ and $\frac{dx^2}{dc^2} = \frac{V''(z^2(c^2))}{V''(z^2(c^2))+U''(x^2(c^2))}$ Taking derivatives and substituting the values of $\frac{dx^2}{dc^2}$ and $\frac{dz^2}{dc^2}$ gives us

$$\frac{d^2z^2}{d(c^2)^2} = \frac{(V'')^2U''' - (U'')^2V'''}{(U'' + V'')^3}$$

Now define H such that $\widetilde{V}(x^2) = V(z^2(x^2)) = H(U(x^2))$. Taking derivatives and using the fact that $\frac{dz^2}{dx^2} = \frac{U''}{V''}$ gives us that

$$H'(U) = \frac{V'U''}{U'V''}.$$

Taking derivatives again gives us that

$$H''(U) = \frac{1}{U'}[\frac{U'V''(V'U''' + V''U''\frac{dz^2}{dx^2}) - V'U''(U'V'''\frac{dz^2}{dx^2} + U''V'')}{(U'V'')^2}].$$

Since $V' = U'$, this reduces to

$$H''(U) = \frac{V'}{V''}[\frac{(V'')^2U''' - (U'')^2V'''}{(U'V'')^2}].$$

Since $(U'' + V'')^3$ has the same sign as V'', the result follows. ■