Chapter 4
Time Value of Money

Solutions to Problems

P4-1. LG 1: Using a Time Line
Basic
(a), (b), and (c)

Compounding

(d) Financial managers rely more on present than future value because they typically make decisions before the start of a project, at time zero, as does the present value calculation.
P4-2. **LG 2: Future Value Calculation: \( FV_n = PV \times (1 + i)^n \)**

**Basic**

**Case**

A  \( FVIF_{12\%, 2 \text{ periods}} = (1 + 0.12)^2 = 1.254 \)

B  \( FVIF_{6\%, 3 \text{ periods}} = (1 + 0.06)^3 = 1.191 \)

C  \( FVIF_{9\%, 2 \text{ periods}} = (1 + 0.09)^2 = 1.188 \)

D  \( FVIF_{3\%, 4 \text{ periods}} = (1 + 0.03)^4 = 1.126 \)

P4-3. **LG 2: Future Value Tables: \( FV_n = PV \times (1 + i)^n \)**

**Basic**

**Case A**

(a)  \( 2 = 1 \times (1 + 0.07)^n \)

\( 2/1 = (1.07)^n \)

\( 2 = FVIF_{7\%, n} \)

10 years < \( n < 11 \) years

Nearest to 10 years

(b)  \( 4 = 1 \times (1 + 0.07)^n \)

\( 4/1 = (1.07)^n \)

\( 4 = FVIF_{7\%, n} \)

20 years < \( n < 21 \) years

Nearest to 20 years

**Case B**

(a)  \( 2 = 1 \times (1 + 0.40)^n \)

\( 2 = FVIF_{40\%, n} \)

2 years < \( n < 3 \) years

Nearest to 2 years

(b)  \( 4 = (1 + 0.40)^n \)

\( 4 = FVIF_{40\%, n} \)

4 years < \( n < 5 \) years

Nearest to 4 years

**Case C**

(a)  \( 2 = 1 \times (1 + 0.20)^n \)

\( 2 = FVIF_{20\%, n} \)

3 years < \( n < 4 \) years

Nearest to 3 years

(b)  \( 4 = (1 + 0.20)^n \)

\( 4 = FVIF_{20\%, n} \)

7 years < \( n < 8 \) years

Nearest to 7 years

**Case D**

(a)  \( 2 = 1 \times (1 + 0.10)^n \)

\( 2 = FVIF_{10\%, n} \)

7 years < \( n < 8 \) years

Nearest to 7 years

(b)  \( 4 = (1 + 0.10)^n \)

\( 4 = FVIF_{10\%, n} \)

14 years < \( n < 15 \) years

Nearest to 14 years

P4-4. **LG 2: Future Values: \( FV_n = PV \times (1 + i)^n \) or \( FV_n = PV \times (FVIF_{i\%, n}) \)**

**Intermediate**

**Case**

A  \( FV_{20} = PV \times FVIF_{5\%, 20 \text{ yrs.}} \)

\( FV_{20} = $200 \times (2.653) \)

\( FV_{20} = $530.60 \)

Calculator solution: $530.66

**Case**

B  \( FV_7 = PV \times FVIF_{8\%, 7 \text{ yrs.}} \)

\( FV_7 = $4,500 \times (1.714) \)

\( FV_7 = $7,713 \)

Calculator solution: $7,712.21
C \[ FV_{10} = PV \times FVIF_{9\%,10\text{ yrs.}} \]
\[ FV_{10} = $10,000 \times (2.367) \]
\[ FV_{10} = $23,670 \]
Calculator solution: $23,673.64

D \[ FV_{12} = PV \times FVIF_{10\%,12\text{ yrs.}} \]
\[ FV_{12} = $25,000 \times (3.138) \]
\[ FV_{12} = $78,450 \]
Calculator solution: $78,460.71

E \[ FV_5 = PV \times FVIF_{11\%,5\text{ yrs.}} \]
\[ FV_5 = $37,000 \times (1.685) \]
\[ FV_5 = $62,345 \]
Calculator solution: $62,347.15

F \[ FV_9 = PV \times FVIF_{12\%,9\text{ yrs.}} \]
\[ FV_9 = $40,000 \times (2.773) \]
\[ FV_9 = $110,920 \]
Calculator solution: $110,923.15

P4-5. LG 2: Time Value: \[ FV_n = PV \times (1 + i)^n \text{ or } FV^n = PV \times (FVIF_{i\%,n}) \]

Intermediate

(a) \[ (1) \quad FV_3 = PV \times (FVIF_{7\%,3}) \]
\[ FV_3 = $1,500 \times (1.225) \]
\[ FV_3 = $1,837.50 \]
Calculator solution: $1,837.56

(b) \[ (1) \quad \text{Interest earned} = FV_3 - PV \]
\[ \text{Interest earned} = $1,837.50 - $1,500.00 \]
\[ \text{Interest earned} = $337.50 \]

(2) \[ (2) \quad FV_6 = PV \times (FVIF_{7\%,6}) \]
\[ FV_6 = $1,500 \times (1.501) \]
\[ FV_6 = $2,251.50 \]
Calculator solution: $2,251.10

(2) \[ (2) \quad \text{Interest earned} = FV_6 - FV_3 \]
\[ \text{Interest earned} = $2,251.50 - $1,837.50 \]
\[ \text{Interest earned} = $414.00 \]

(3) \[ (3) \quad FV_9 = PV \times (FVIF_{7\%,9}) \]
\[ FV_9 = $1,500 \times (1.838) \]
\[ FV_9 = $2,757.00 \]
Calculator solution: $2,757.69

(3) \[ (3) \quad \text{Interest earned} = FV_9 - FV_6 \]
\[ \text{Interest earned} = $2,757.00 - $2,251.50 \]
\[ \text{Interest earned} = $505.50 \]

(c) The fact that the longer the investment period is, the larger the total amount of interest collected will be, is not unexpected and is due to the greater length of time that the principal sum of $1,500 is invested. The most significant point is that the incremental interest earned per 3-year period increases with each subsequent 3-year period. The total interest for the first 3 years is $337.50; however, for the second 3 years (from year 3 to 6) the additional interest earned is $414.00. For the third 3-year period, the incremental interest is $505.50. This increasing change in interest earned is due to compounding, the earning of interest on previous interest earned. The greater the previous interest earned, the greater the impact of compounding.
P4-6. LG 2: Time Value
Challenge
(a) \( FV_5 = PV \times (FVIF_{2\%,5}) \)
\[
FV_5 = $14,000 \times (1.104) = \text{FV}_5 = $15,456.00
\]
Calculator solution: $15,457.13
(b) The car will cost $1,582 more with a 4% inflation rate than an inflation rate of 2%. This
increase is 10.2% more ($1,582 ÷ $15,456) than would be paid with only a 2% rate of
inflation.

P4-7. LG 2: Time Value
Challenge
Deposit now:
\[
FV_{40} = PV \times FVIF_{9\%,40}
\]
\[
FV_{40} = $10,000 \times (1.09)^{40} = \text{FV}_40 = $314,090.00
\]
Calculator solution: $314,094.20
Deposit in 10 years:
\[
FV_{30} = PV_{10} \times (FVIF_{9\%,30})
\]
\[
FV_{30} = $10,000 \times (1.09)^{30} = \text{FV}_30 = $132,680.00
\]
Calculator solution: $132,676.79
You would be better off by $181,410 ($314,090 – $132,680) by investing the $10,000 now instead
of waiting for 10 years to make the investment.

P4-8. LG 2: Time Value: \( FV_n = PV \times FVIF_{i\%,n} \)
Challenge
(a) \$15,000 = $10,200 \times FVIF_{9\%,5}
\[
FVIF_{9\%,5} = $15,000 \div $10,200 = 1.471
\]
8% < i < 9%
Calculator Solution: 8.02%
(b) \$15,000 = $8,150 \times FVIF_{13\%,5}
\[
FVIF_{13\%,5} = $15,000 \div $8,150 = 1.840
\]
12% < i < 13%
Calculator Solution: 12.98%
(c) \$15,000 = $7,150 \times FVIF_{16\%,5}
\[
FVIF_{16\%,5} = $15,000 \div $7,150 = 2.098
\]
15% < i < 16%
Calculator Solution: 15.97%
P4-9. LG 2: Single-payment Loan Repayment: \( FV_n = PV \times FVIF_{i\%,n} \)

Intermediate

\( (a) \quad FV_1 = PV \times (FVIF_{14\%,1}) \)
\( \quad FV_1 = \$200 \times (1.14) \)
\( \quad FV_1 = \$228 \)
\( \quad \text{Calculator Solution:} \$228 \)

\( (b) \quad FV_4 = PV \times (FVIF_{14\%,4}) \)
\( \quad FV_4 = \$200 \times (1.689) \)
\( \quad FV_4 = \$337.80 \)
\( \quad \text{Calculator solution:} \$337.79 \)

\( (c) \quad FV_8 = PV \times (FVIF_{14\%,8}) \)
\( \quad FV_8 = \$200 \times (2.853) \)
\( \quad FV_8 = \$570.60 \)
\( \quad \text{Calculator Solution:} \$570.52 \)

P4-10. LG 2: Present Value Calculation: \( PV = \frac{1}{(1 + i)^n} \)

Basic Case

A \( PV = \frac{1}{(1 + 0.02)^4} = 0.9238 \)
B \( PV = \frac{1}{(1 + 0.10)^2} = 0.8264 \)
C \( PV = \frac{1}{(1 + 0.05)^3} = 0.8638 \)
D \( PV = \frac{1}{(1 + 0.13)^2} = 0.7831 \)

P4-11. LG 2: Present Values: \( PV = FV_n \times (PVIF_{i\%,n}) \)

Basic Case

A \( PV_{12\%,4yrs} = \$7,000 \times 0.636 = \$4,452 \) \( \quad \text{Calculator Solution} = \$4,448.63 \)
B \( PV_{8\%,20yrs} = \$28,000 \times 0.215 = \$6,020 \) \( \quad \text{Calculator Solution} = \$6,007.35 \)
C \( PV_{14\%,12yrs} = \$10,000 \times 0.208 = \$2,080 \) \( \quad \text{Calculator Solution} = \$2,075.59 \)
D \( PV_{11\%,6yrs} = \$150,000 \times 0.535 = \$80,250 \) \( \quad \text{Calculator Solution} = \$80,196.13 \)
E \( PV_{20\%,8yrs} = \$45,000 \times 0.233 = \$10,485 \) \( \quad \text{Calculator Solution} = \$10,465.56 \)

P4-12. LG 2: Present Value Concept: \( PV_n = FV_n \times (PVIF_{i\%,n}) \)

Intermediate Case

\( (a) \quad PV = FV_6 \times (PVIF_{12\%,6}) \)
\( \quad PV = \$6,000 \times (.507) \)
\( \quad PV = \$3,042.00 \)
\( \quad \text{Calculator solution:} \$3,039.79 \)

\( (b) \quad PV = FV_6 \times (PVIF_{12\%,6}) \)
\( \quad PV = \$6,000 \times (0.507) \)
\( \quad PV = \$3,042.00 \)
\( \quad \text{Calculator solution:} \$3,039.79 \)

\( (c) \quad PV = FV_6 \times (PVIF_{12\%,6}) \)
\( \quad PV = \$6,000 \times (0.507) \)
\( \quad PV = \$3,042.00 \)
\( \quad \text{Calculator solution:} \$3,039.79 \)

\( (d) \quad \text{The answer to all three parts are the same. In each case the same questions is being asked but in a different way.} \)
P4-13. LG 2: Time Value: \( PV = FV_n \times (PVIF_{i%,n}) \)

Basic

Jim should be willing to pay no more than $408.00 for this future sum given that his opportunity cost is 7%.

\[
PV = $500 \times (PVIF_{7\%,3})
\]

\[
PV = $500 \times (0.816)
\]

\[
PV = $408.00
\]

Calculator solution: $408.15

P4-14. LG 2: Time Value: \( PV = FV_n \times (PVIF_{i%,n}) \)

Intermediate

\[
PV = $100 \times (PVIF_{8\%,6})
\]

\[
PV = $100 \times (0.630)
\]

\[
PV = $63.00
\]

Calculator solution: $63.02

P4-15. LG 2: Time Value and Discount Rates: \( PV = FV_n \times (PVIF_{i%,n}) \)

Intermediate

(a)  (1) \( PV = $1,000,000 \times (PVIF_{6\%,10}) \)

\[
PV = $1,000,000 \times (0.558)
\]

\[
PV = $558,000.00
\]

Calculator solution: $558,394.78

(2) \( PV = $1,000,000 \times (PVIF_{9\%,10}) \)

\[
PV = $1,000,000 \times (0.422)
\]

\[
PV = $422,000.00
\]

Calculator solution: $422,410.81

(3) \( PV = $1,000,000 \times (PVIF_{12\%,10}) \)

\[
PV = $1,000,000 \times (0.322)
\]

\[
PV = $322,000.00
\]

Calculator solution: $321,973.24

(b)  (1) \( PV = $1,000,000 \times (PVIF_{6\%,15}) \)

\[
PV = $1,000,000 \times (0.417)
\]

\[
PV = $417,000.00
\]

Calculator solution: $417,265.06

(2) \( PV = $1,000,000 \times (PVIF_{9\%,15}) \)

\[
PV = $1,000,000 \times (0.275)
\]

\[
PV = $275,000.00
\]

Calculator solution: $274,538.04

(3) \( PV = $1,000,000 \times (PVIF_{12\%,15}) \)

\[
PV = $1,000,000 \times (0.183)
\]

\[
PV = $183,000.00
\]

Calculator solution: $182,696.26

(c) As the discount rate increases, the present value becomes smaller. This decrease is due to the higher opportunity cost associated with the higher rate. Also, the longer the time until the lottery payment is collected, the less the present value due to the greater time over which the opportunity cost applies. In other words, the larger the discount rate and the longer the time until the money is received, the smaller will be the present value of a future payment.
P4-16. LG 2: Time Value Comparisons of Lump Sums: \( PV = FV_n \times (PVIF_{i%,n}) \)

Intermediate

(a) A
\[
PV = \$28,500 \times (PVIF_{11\%,3}) \\
PV = \$28,500 \times (0.731) \\
PV = \$20,833.50 \\
Calculator solution: $20,838.95
\]

B
\[
PV = \$54,000 \times (PVIF_{11\%,9}) \\
PV = \$54,000 \times (0.391) \\
PV = \$21,114.00 \\
Calculator solution: $21,109.94
\]

C
\[
PV = \$160,000 \times (PVIF_{11\%,20}) \\
PV = \$160,000 \times (0.124) \\
PV = \$19,840.00 \\
Calculator solution: $19,845.43
\]

(b) Alternatives A and B are both worth greater than $20,000 in term of the present value.

(c) The best alternative is B because the present value of B is larger than either A or C and is also greater than the $20,000 offer.

P4-17. LG 2: Cash Flow Investment Decision: \( PV = FV_n \times (PVIF_{i%,n}) \)

Intermediate

A
\[
PV = \$30,000 \times (PVIF_{10\%,5}) \\
PV = \$30,000 \times (0.621) \\
PV = \$18,630.00 \\
Calculator solution: $18,627.64
\]

B
\[
PV = \$3,000 \times (PVIF_{10\%,20}) \\
PV = \$3,000 \times (0.149) \\
PV = \$447.00 \\
Calculator solution: $445.93
\]

C
\[
PV = \$10,000 \times (PVIF_{10\%,10}) \\
PV = \$10,000 \times (0.386) \\
PV = \$3,860.00 \\
Calculator solution: $3,855.43
\]

D
\[
PV = \$15,000 \times (PVIF_{10\%,40}) \\
PV = \$15,000 \times (0.022) \\
PV = \$330.00 \\
Calculator solution: $331.42
\]

<table>
<thead>
<tr>
<th>Purchase</th>
<th>Do Not Purchase</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>C</td>
<td>D</td>
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</table>
P4-18. LG 3: Future Value of an Annuity  
Intermediate  
(a) Future Value of an Ordinary Annuity vs. Annuity Due  
(1) **Ordinary Annuity**  
\[ FVA_{k\%,n} = PMT \times (FVIFA_{k\%,n}) \]  
(2) **Annuity Due**  
\[ FVA_{due} = PMT \times [(FVIFA_{k\%,n} \times (1 + k))] \]  

<table>
<thead>
<tr>
<th>Case</th>
<th>Ordinary Annuity</th>
<th>Annuity Due</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>[ FVA_{8%,10} = 2,500 \times 14.487 ]</td>
<td>[ FVA_{due} = 2,500 \times (14.487 \times 1.08) ]</td>
</tr>
<tr>
<td></td>
<td>[ FVA_{8%,10} = 36,217.50 ]</td>
<td>[ FVA_{due} = 39,114.90 ]</td>
</tr>
<tr>
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<td>Calculator solution: $36,216.41</td>
<td>Calculator solution: $39,113.72</td>
</tr>
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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>[ FVA_{12%,6} = 500 \times 8.115 ]</td>
<td>[ FVA_{due} = 500 \times (8.115 \times 1.12) ]</td>
</tr>
<tr>
<td></td>
<td>[ FVA_{12%,6} = 4,057.50 ]</td>
<td>[ FVA_{due} = 4,544.40 ]</td>
</tr>
<tr>
<td></td>
<td>Calculator solution: $4,057.59</td>
<td>Calculator solution: $4,544.51</td>
</tr>
</tbody>
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<tr>
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</thead>
<tbody>
<tr>
<td>C</td>
<td>[ FVA_{20%,5} = 30,000 \times 7.442 ]</td>
<td>[ FVA_{due} = 30,000 \times (7.442 \times 1.20) ]</td>
</tr>
<tr>
<td></td>
<td>[ FVA_{20%,5} = 223,260 ]</td>
<td>[ FVA_{due} = 267,912 ]</td>
</tr>
<tr>
<td></td>
<td>Calculator solution: $223,248</td>
<td>Calculator solution: $267,897.60</td>
</tr>
</tbody>
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<thead>
<tr>
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<tbody>
<tr>
<td>D</td>
<td>[ FVA_{9%,8} = 11,500 \times 11.028 ]</td>
<td>[ FVA_{due} = 11,500 \times (11.028 \times 1.09) ]</td>
</tr>
<tr>
<td></td>
<td>[ FVA_{9%,8} = 126,822 ]</td>
<td>[ FVA_{due} = 138,235.98 ]</td>
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<tr>
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</thead>
<tbody>
<tr>
<td>E</td>
<td>[ FVA_{14%,30} = 6,000 \times 356.787 ]</td>
<td>[ FVA_{due} = 6,000 \times (356.787 \times 1.14) ]</td>
</tr>
<tr>
<td></td>
<td>[ FVA_{14%,30} = 2,140,722 ]</td>
<td>[ FVA_{due} = 2,440,422.00 ]</td>
</tr>
<tr>
<td></td>
<td>Calculator solution: $2,140,721.10</td>
<td>Calculator solution: $2,440,422.03</td>
</tr>
</tbody>
</table>

(b) The annuity due results in a greater future value in each case. By depositing the payment at the beginning rather than at the end of the year, it has one additional year of compounding.

P4-19. LG 3: Present Value of an Annuity:  
\[ PV_{n} = PMT \times (PVIFA_{r\%,n}) \]  
Intermediate  
(a) Present Value of an Ordinary Annuity vs. Annuity Due  
(1) **Ordinary Annuity**  
\[ PVA_{r\%,n} = PMT \times (PVIFA_{r\%,n}) \]  
(2) **Annuity Due**  
\[ PVA_{due} = PMT \times [(PVIFA_{r\%,n} \times (1 + k))] \]  

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<tr>
<th>Case</th>
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</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>[ PVA_{7%,3} = 12,000 \times 2.624 ]</td>
<td>[ PVA_{due} = 12,000 \times (2.624 \times 1.07) ]</td>
</tr>
<tr>
<td></td>
<td>[ PVA_{7%,3} = 31,488 ]</td>
<td>[ PVA_{due} = 33,692 ]</td>
</tr>
<tr>
<td></td>
<td>Calculator solution: $31,491.79</td>
<td>Calculator solution: $33,692.16</td>
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<thead>
<tr>
<th>Case</th>
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</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>[ PVA_{12%,15} = 55,000 \times 6.811 ]</td>
<td>[ PVA_{due} = 55,000 \times (6.811 \times 1.12) ]</td>
</tr>
<tr>
<td></td>
<td>[ PVA_{12%,15} = 374,605 ]</td>
<td>[ PVA_{due} = 419,557.60 ]</td>
</tr>
<tr>
<td></td>
<td>Calculator solution: $374,597.55</td>
<td>Calculator solution: $419,549.25</td>
</tr>
</tbody>
</table>
C  \( PVA_{20\%,9} = \$700 \times 4.031 \)  \( PVA_{due} = \$700 \times (4.031 \times 1.20) \)
\( PVA_{20\%,9} = \$2,821.70 \)  \( PVA_{due} = \$3,386.04 \)
Calculator solution: $2,821.68  Calculator solution: $3,386.01

D  \( PVA_{5\%,7} = \$140,000 \times 5.786 \)  \( PVA_{due} = \$140,000 \times (5.786 \times 1.05) \)
\( PVA_{5\%,7} = \$810,040 \)  \( PVA_{due} = \$850,542 \)
Calculator solution: $810,092.28  Calculator solution: $850,596.89

E  \( PVA_{10\%,5} = \$22,500 \times 3.791 \)  \( PVA_{due} = \$22,500 \times (2.791 \times 1.10) \)
\( PVA_{10\%,5} = \$85,297.50 \)  \( PVA_{due} = \$93,827.25 \)
Calculator solution: $85,292.70  Calculator solution: $93,821.97

(b) The annuity due results in a greater present value in each case. By depositing the payment at the beginning rather than at the end of the year, it has one less year to discount back.

P4-20. LG 3: Time Value–Annuities
Challenge

(a) **Annuity C (Ordinary)**  **Annuity D (Due)**
\( FVA_{i\%,n} = PMT \times (FVIFA_{i\%,n}) \)  \( FVA_{due} = PMT \times [FVIFA_{i\%,n} \times (1 + i)] \)
(1)  \( FVA_{10\%,10} = \$2,500 \times 15.937 \)  \( FVA_{due} = \$2,200 \times (15.937 \times 1.10) \)
\( FVA_{10\%,10} = \$39,842.50 \)  \( FVA_{due} = \$38,567.54 \)
Calculator solution: $39,843.56  Calculator solution: $38,568.57
(2)  \( FVA_{20\%,10} = \$2,500 \times 25.959 \)  \( FVA_{due} = \$2,200 \times (25.959 \times 1.20) \)
\( FVA_{20\%,10} = \$64,897.50 \)  \( FVA_{due} = \$68,531.76 \)
Calculator solution: $64,896.71  Calculator solution: $68,530.92

(b) (1) At the end of year 10, at a rate of 10%, Annuity C has a greater value ($39,842.50 vs. $38,567.54).
(2) At the end of year 10, at a rate of 20%, Annuity D has a greater value ($68,531.76 vs. $64,896.71).

(c) **Annuity C (Ordinary)**  **Annuity D (Due)**
\( PVA_{i\%,n} = PMT \times (FVIFA_{i\%,n}) \)  \( PVA_{due} = PMT \times [FVIFA_{i\%,n} \times (1 + i)] \)
(1)  \( PVA_{10\%,10} = \$2,500 \times 6.145 \)  \( PVA_{due} = \$2,200 \times (6.145 \times 1.10) \)
\( PVA_{10\%,10} = \$15,362.50 \)  \( PVA_{due} = \$14,870.90 \)
Calculator solution: $15,361.42  Calculator solution: $14,869.85
(2)  \( PVA_{20\%,10} = \$2,500 \times 4.192 \)  \( PVA_{due} = \$2,200 \times (4.192 \times 1.20) \)
\( PVA_{20\%,10} = \$10,480 \)  \( PVA_{due} = \$11,066.88 \)
Calculator solution: $10,481.18  Calculator solution: $11,068.13
(d) (1) At the beginning of the 10 years, at a rate of 10%, Annuity C has a greater value ($15,362.50 vs. $14,870.90).
(2) At the beginning of the 10 years, at a rate of 20%, Annuity D has a greater value ($11,066.88 vs. $10,480.00).
(e) Annuity C, with an annual payment of $2,500 made at the end of the year, has a higher present value at 10% than Annuity D with an annual payment of $2,200 made at the beginning of the year. When the rate is increased to 20%, the shorter period of time to discount at the higher rate results in a larger value for Annuity D, despite the lower payment.

P4-21. LG 3: Retirement Planning

Challenge

(a) FVA\(_{40}\) = $2,000 \times (FVIFA_{10\%,40})
FVA\(_{40}\) = $2,000 \times (442.593)
FVA\(_{40}\) = $885,186
Calculator solution: $885,186.11
Calculator solution: $885,186.11

(b) FVA\(_{30}\) = $2,000 \times (FVIFA_{10\%,30})
FVA\(_{30}\) = $2,000 \times (164.494)
FVA\(_{30}\) = $328,988
Calculator solution: $328,988.05
Calculator solution: $328,988.05

(c) By delaying the deposits by 10 years the total opportunity cost is $556,198. This difference is due to both the lost deposits of $20,000 ($2,000 \times 10\text{ yrs.}) and the lost compounding of interest on all of the money for 10 years.

(d) **Annuity Due:**
FVA\(_{40}\) = $2,000 \times (FVIFA_{10\%,40}) \times (1 + 0.10)
FVA\(_{40}\) = $2,000 \times (486.852)
FVA\(_{40}\) = $973,704
Calculator solution: $973,703.62

FVA\(_{30}\) = $2,000 \times (FVIFA_{10\%,30}) \times (1.10)
FVA\(_{30}\) = $2,000 \times (180.943)
FVA\(_{30}\) = $361,886
Calculator solution: $361,886.85

Both deposits increased due to the extra year of compounding from the beginning-of-year deposits instead of the end-of-year deposits. However, the incremental change in the 40 year annuity is much larger than the incremental compounding on the 30 year deposit ($88,518 versus $32,898) due to the larger sum on which the last year of compounding occurs.

P4-22. LG 3: Value of a Retirement Annuity

Intermediate

PVA = PMT \times (PVIFA_{9\%,25})
PVA = $12,000 \times (9.823)
PVA = $117,876.00
Calculator solution: $117,870.96
P4-23. LG 3: Funding Your Retirement

**Challenge**

(a) \( PVA = PMT \times (PVIFA_{11\%,30}) \)

\[ PVA = 20,000 \times (8.694) \]

\[ PVA = 173,880 \]

Calculator solution: \( 173,875.85 \)

(b) \( PV = FV \times (PVIF_{9\%,20}) \)

\[ PV = 30,950.64 \]

Calculator solution: \( 31,024.82 \)

(c) Both values would be lower. In other words, a smaller sum would be needed in 20 years for the annuity and a smaller amount would have to be put away today to accumulate the needed future sum.

P4-24. LG 2, 3: Value of an Annuity versus a Single Amount

**Intermediate**

(a) \( PVA_n = PMT \times (PVIFA_{i\%,n}) \)

\[ PVA_{25} = 40,000 \times (PVIFA_{5\%,25}) \]

\[ PVA_{25} = 40,000 \times 14.094 \]

\[ PVA_{25} = 563,760 \]

Calculator solution: \( 563,757.78 \)

At 5%, taking the award as an annuity is better; the present value is \( 563,760 \), compared to receiving \( 500,000 \) as a lump sum.

(b) \( PVA_n = PMT \times (PVIFA_{7\%,25}) \)

\[ PVA_{25} = 40,000 \times (11.654) \]

\[ PVA_{25} = 466,160 \]

Calculator solution: \( 466,143.33 \)

At 7%, taking the award as a lump sum is better; the present value of the annuity is only \( 466,160 \), compared to the \( 500,000 \) lump sum payment.

(c) Because the annuity is worth more than the lump sum at 5% and less at 7%, try 6%:

\[ PV_{25} = 40,000 \times (PVIFA_{6\%,25}) \]

\[ PV_{25} = 40,000 \times 12.783 \]

\[ PV_{25} = 511,320 \]

The rate at which you would be indifferent is greater than 6%; about 6.25% Calculator solution: 6.24%

P4-25. LG 3: Perpetuities: \( PV_n = PMT \times (PVIFA_{i\%\infty}) \)

**Basic**

(a) \( PMT \times (PVIFA_{i\%\infty}) = PMT \times (1 + i) \)

<table>
<thead>
<tr>
<th>Case</th>
<th>PV Factor</th>
<th>(b) ( PMT \times (PVIFA_{i%\infty}) = PMT \times (1 + i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1 ÷ 0.08 = 12.50</td>
<td>$20,000 \times 12.50 = 250,000</td>
</tr>
<tr>
<td>B</td>
<td>1 ÷ 0.10 = 10.00</td>
<td>$100,000 \times 10.00 = 1,000,000</td>
</tr>
<tr>
<td>C</td>
<td>1 ÷ 0.06 = 16.67</td>
<td>$3,000 \times 16.67 = 50,000</td>
</tr>
<tr>
<td>D</td>
<td>1 ÷ 0.05 = 20.00</td>
<td>$60,000 \times 20.00 = 1,200,000</td>
</tr>
</tbody>
</table>
P4-26. LG 3: Creating an Endowment
Intermediate

(a) \( PV = PMT \times (PVIFA_{i\%/n}) \)
\[
PV = ($600 \times 3) \times (1 \div i)
\]
\[
PV = $1,800 \times (1 \div i)
\]
\[
PV = $1,800 \times (16.67)
\]
\[
PV = $30,006
\]

(b) \( PV = PMT \times (PVIFA_{i\%/\infty}) \)
\[
PV = ($600 \times 3) \times (1 \div i)
\]
\[
PV = $1,800 \times (1 \div 0.06)
\]
\[
PV = $1,800 \times (11.11)
\]
\[
PV = $30,006
\]

P4-27. LG 4: Value of a Mixed Stream
Challenge

(a)

<table>
<thead>
<tr>
<th>Cash Flow Stream</th>
<th>Year</th>
<th>Number of Years to Compound</th>
<th>( FV = CF \times FVIF_{12%,n} )</th>
<th>Future Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>3</td>
<td>$900 \times 1.405 =</td>
<td>$1,264.50</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2</td>
<td>$1,000 \times 1.254 =</td>
<td>$1,254.00</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1</td>
<td>$1,200 \times 1.120 =</td>
<td>$1,344.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>$3,862.50</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Calculator Solution:</td>
<td>$3,862.84</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>5</td>
<td>$30,000 \times 1.762 =</td>
<td>$52,860.00</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4</td>
<td>$25,000 \times 1.574 =</td>
<td>$39,350.00</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3</td>
<td>$20,000 \times 1.405 =</td>
<td>$28,100.00</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>2</td>
<td>$10,000 \times 1.254 =</td>
<td>$12,540.00</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>1</td>
<td>$5,000 \times 1.120 =</td>
<td>$5,600.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Calculator Solution:</td>
<td><strong>$138,450.00</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$138,450.79.</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>4</td>
<td>$1,200 \times 1.574 =</td>
<td>$1,888.80</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3</td>
<td>$1,200 \times 1.405 =</td>
<td>$1,686.00</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2</td>
<td>$1,000 \times 1.254 =</td>
<td>$1,254.00</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1</td>
<td>$1,900 \times 1.120 =</td>
<td>$2,128.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Calculator Solution:</td>
<td><strong>$6,956.80</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$6,956.53.</td>
<td></td>
</tr>
</tbody>
</table>
(b) If payments are made at the beginning of each period the present value of each of the end-of-period cash flow streams will be multiplied by \((1 + i)\) to get the present value of the beginning-of-period cash flows.

\[
\begin{align*}
\text{A} & \quad \$3,862.50 \times (1 + 0.12) = \$4,326.00 \\
\text{B} & \quad \$138,450.00 \times (1 + 0.12) = \$155,064.00 \\
\text{C} & \quad \$6,956.80 \times (1 + 0.12) = \$7,791.62
\end{align*}
\]

P4-28. LG 4: Value of a Single Amount Versus a Mixed Stream

Intermediate

**Lump Sum Deposit**

\[
FV_5 = PV \times (FVIF_{7\%}, 5)
\]

\[
FV_5 = \$24,000 \times (1.403)
\]

\[
FV_5 = \$33,672.00
\]

Calculator solution: \$33,661.24

**Mixed Stream of Payments**

<table>
<thead>
<tr>
<th>Beginning of Year</th>
<th>Number of Years to Compound</th>
<th>[FV = CF \times FVIF_{7%},n]</th>
<th>Future Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>$2,000 \times 1.403 =</td>
<td>$2,806.00</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>$4,000 \times 1.311 =</td>
<td>$5,244.00</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>$6,000 \times 1.225 =</td>
<td>$7,350.00</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>$8,000 \times 1.145 =</td>
<td>$9,160.00</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>$10,000 \times 1.070 =</td>
<td>$10,700.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$35,260.00</td>
</tr>
</tbody>
</table>

Calculator Solution: \$35,257.74

Gina should select the stream of payments over the front-end lump sum payment. Her future wealth will be higher by \$1,588.
### P4-29. LG 4: Value of Mixed Stream

#### Basic

<table>
<thead>
<tr>
<th>Cash Flow Stream</th>
<th>Year</th>
<th>CF</th>
<th>× PVIF&lt;sub&gt;12%,n&lt;/sub&gt;</th>
<th>= Present Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td>1</td>
<td>−$2000</td>
<td>0.893</td>
<td>−$1,786</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3,000</td>
<td>0.797</td>
<td>2,391</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4,000</td>
<td>0.712</td>
<td>2,848</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>6,000</td>
<td>0.636</td>
<td>3,816</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>8,000</td>
<td>0.567</td>
<td>4,536</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>$11,805</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Calculator solution: <strong>$11,805.51</strong></td>
</tr>
<tr>
<td><strong>B</strong></td>
<td>1</td>
<td>$10,000</td>
<td>0.893</td>
<td>$8,930</td>
</tr>
<tr>
<td></td>
<td>2–5</td>
<td>5,000</td>
<td>2.712&lt;sup&gt;*&lt;/sup&gt;</td>
<td>13,560</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>7,000</td>
<td>0.507</td>
<td>3,549</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>$26,039</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Calculator solution: <strong>$26,034.59</strong></td>
</tr>
</tbody>
</table>

<sup>*</sup> Sum of PV factors for years 2–5

| **C**            | 1–5  | −$10,000 | 3.605<sup>*</sup> | $36,050        |
|                  | 6–10 | 8,000   | 2.045<sup>**</sup> | 16,360         |
|                  |      |         |                        | **$52,410**    |
|                  |      |         |                        | Calculator Solution: **$52,411.34** |

<sup>*</sup> PVIFA for 12% 5 years

<sup>**</sup> (PVIFA for 12%,10 years) – (PVIFA for 12%,5 years)
P4-30. LG 4: Present Value-Mixed Stream
Intermediate
(a)  

<table>
<thead>
<tr>
<th>Cash Flow Stream</th>
<th>Year</th>
<th>CF</th>
<th>PVIF_{15%,n}</th>
<th>Present Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>$50,000</td>
<td>0.870</td>
<td>$43,500</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>40,000</td>
<td>0.756</td>
<td>30,240</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>30,000</td>
<td>0.658</td>
<td>19,740</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>20,000</td>
<td>0.572</td>
<td>11,440</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>10,000</td>
<td>0.497</td>
<td>4,970</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>$109,890</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Calculator solution <strong>$109,856.33</strong></td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>$10,000</td>
<td>0.870</td>
<td>$8,700</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>20,000</td>
<td>0.756</td>
<td>15,120</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>30,000</td>
<td>0.658</td>
<td>19,740</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>40,000</td>
<td>0.572</td>
<td>22,880</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>50,000</td>
<td>0.497</td>
<td>24,850</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>$91,290</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Calculator solution <strong>$91,272.98</strong></td>
</tr>
</tbody>
</table>

(b) Cash flow stream A, with a present value of $109,890, is higher than cash flow stream B’s present value of $91,290 because the larger cash inflows occur in A in the early years when their present value is greater, while the smaller cash flows are received further in the future.

P4-31. LG 1, 4: Value of a Mixed Stream
Intermediate
(a)  

<table>
<thead>
<tr>
<th>Cash Flows</th>
</tr>
</thead>
<tbody>
<tr>
<td>$30,000</td>
</tr>
<tr>
<td>$25,000</td>
</tr>
<tr>
<td>$15,000</td>
</tr>
<tr>
<td>$15,000</td>
</tr>
<tr>
<td>$15,000</td>
</tr>
<tr>
<td>$10,000</td>
</tr>
</tbody>
</table>

End of Year
### Part 2: Important Financial Concepts

(b)

<table>
<thead>
<tr>
<th>Cash Flow Stream</th>
<th>Year</th>
<th>CF</th>
<th>PVIF(_{12%,n})</th>
<th>Present Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>$30,000</td>
<td>0.893</td>
<td>$26,790</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$25,000</td>
<td>0.797</td>
<td>19,925</td>
</tr>
<tr>
<td></td>
<td>3–9</td>
<td>$15,000</td>
<td>3.639(^*)</td>
<td>54,585</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>$10,000</td>
<td>0.322</td>
<td>3,220</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$104,520</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Calculator solution</td>
</tr>
</tbody>
</table>

\(^*\) The PVIF for this 7-year annuity is obtained by summing together the PVIFs of 12% for periods 3 through 9. This factor can also be calculated by taking the PVIFA\(_{12\%,7}\) and multiplying by the PVIF\(_{12\%,2}\).

(c) Harte should accept the series of payments offer. The present value of that mixed stream of payments is greater than the $100,000 immediate payment.

P4-32. LG 5: Funding Budget Shortfalls

Intermediate

(a)

<table>
<thead>
<tr>
<th>Year</th>
<th>Budget Shortfall</th>
<th>PVIF(_{8%,n})</th>
<th>Present Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$5,000</td>
<td>0.926</td>
<td>$4,630</td>
</tr>
<tr>
<td>2</td>
<td>4,000</td>
<td>0.857</td>
<td>3,428</td>
</tr>
<tr>
<td>3</td>
<td>6,000</td>
<td>0.794</td>
<td>4,764</td>
</tr>
<tr>
<td>4</td>
<td>10,000</td>
<td>0.735</td>
<td>7,350</td>
</tr>
<tr>
<td>5</td>
<td>3,000</td>
<td>0.681</td>
<td>2,043</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$22,215</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Calculator solution: $22,214.03</td>
</tr>
</tbody>
</table>

A deposit of $22,215 would be needed to fund the shortfall for the pattern shown in the table.

(b) An increase in the earnings rate would reduce the amount calculated in part (a). The higher rate would lead to a larger interest being earned each year on the investment. The larger interest amounts will permit a decrease in the initial investment to obtain the same future value available for covering the shortfall.
P4-33. LG 4: Relationship between Future Value and Present Value-Mixed Stream

Intermediate

(a) **Present Value**

<table>
<thead>
<tr>
<th>Year</th>
<th>CF</th>
<th>( \times )</th>
<th>PVIF(_{5%,\text{n}} )</th>
<th>=</th>
<th>Present Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$800</td>
<td>\times</td>
<td>0.952</td>
<td>=</td>
<td>$761.60</td>
</tr>
<tr>
<td>2</td>
<td>900</td>
<td>\times</td>
<td>0.907</td>
<td>=</td>
<td>816.30</td>
</tr>
<tr>
<td>3</td>
<td>1,000</td>
<td>\times</td>
<td>0.864</td>
<td>=</td>
<td>864.00</td>
</tr>
<tr>
<td>4</td>
<td>1,500</td>
<td>\times</td>
<td>0.822</td>
<td>=</td>
<td>1,233.00</td>
</tr>
<tr>
<td>5</td>
<td>2,000</td>
<td>\times</td>
<td>0.784</td>
<td>=</td>
<td>1,568.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>$5,242.90</strong></td>
</tr>
</tbody>
</table>

Calculator Solution: **$5,243.17**

(b) The maximum you should pay is $5,242.90.
(c) A higher 7% discount rate will cause the present value of the cash flow stream to be lower than $5,242.90.

P4-34. LG 5: Changing Compounding Frequency

Intermediate

(1) Compounding Frequency: \( FV_n = PV \times FVIF_{i\%/m,n,m} \)

(a) **Annual**

12%, 5 years
\[ FV_5 = $5,000 \times (1.762) \]
\[ FV_5 = $8,810 \]
Calculator solution: $8,811.71

Semiannual
12% \div 2 = 6%, 5 \times 2 = 10 periods
\[ FV_5 = $5,000 \times (1.791) \]
\[ FV_5 = $8,955 \]
Calculator solution: $8,954.24

Quarterly
12% \div 4 = 3%, 5 \times 4 = 20 periods
\[ FV_5 = $5,000 \times (1.806) \]
\[ FV_5 = $9,030 \]
Calculator solution: $9,030.56

(b) **Annual**

16%, 6 years
\[ FV_6 = $5,000 \times (2.436) \]
\[ FV_6 = $12,180 \]
Calculator solution: $12,181.98

Semiannual
16% \div 2 = 8%, 6 \times 2 = 12 periods
\[ FV_6 = $5,000 \times (2.518) \]
\[ FV_6 = $12,590 \]
Calculator solution: $12,590.85

Quarterly
16% \div 4 = 4%, 6 \times 4 = 24 periods
\[ FV_6 = $5,000 \times (2.563) \]
\[ FV_6 = $12,815 \]
Calculator solution: $12,816.52
(c) **Annual**

20%, 10 years

\[ FV_{10} = \$5,000 \times (6.192) \]

\[ FV_{10} = \$30,960 \]

Calculator solution: $30,958.68

**Semiannual**

\[ 20\% \div 2 = 10\%, \ 10 \times 2 = 20 \text{ periods} \]

\[ FV_{10} = \$5,000 \times (6.727) \]

\[ FV_{10} = \$33,635 \]

Calculator solution: $33,637.50

**Quarterly**

20% \div 4 = 5%, 10 \times 4 = 40 periods

\[ FV_{10} = \$5,000 \times (7.040) \]

\[ FV_{10} = \$35,200 \]

Calculator solution: $35,199.94

(2) Effective Interest Rate: \( i_{\text{eff}} = (1 + i/m)^m - 1 \)

(a) **Annual**

\[ i_{\text{eff}} = (1 + 0.12/1)^1 - 1 \]

\[ i_{\text{eff}} = (1.12)^1 - 1 \]

\[ i_{\text{eff}} = 0.12 = 12\% \]

**Semiannual**

\[ i_{\text{eff}} = (1 + 0.12/2)^2 - 1 \]

\[ i_{\text{eff}} = (1.06)^2 - 1 \]

\[ i_{\text{eff}} = 0.124 = 12.4\% \]

**Quarterly**

\[ i_{\text{eff}} = (1 + 12/4)^4 - 1 \]

\[ i_{\text{eff}} = (1.03)^4 - 1 \]

\[ i_{\text{eff}} = 0.126 = 12.6\% \]

(b) **Annual**

\[ i_{\text{eff}} = (1 + 0.16/1)^1 - 1 \]

\[ i_{\text{eff}} = (1.16)^1 - 1 \]

\[ i_{\text{eff}} = 0.16 = 16\% \]

**Semiannual**

\[ i_{\text{eff}} = (1 + 0.16/2)^2 - 1 \]

\[ i_{\text{eff}} = (1.08)^2 - 1 \]

\[ i_{\text{eff}} = 0.166 = 16.6\% \]

**Quarterly**

\[ i_{\text{eff}} = (1 + 0.16/4)^4 - 1 \]

\[ i_{\text{eff}} = (1.04)^4 - 1 \]

\[ i_{\text{eff}} = 0.170 = 17\% \]

(c) **Annual**

\[ i_{\text{eff}} = (1 + 0.20/1)^1 - 1 \]

\[ i_{\text{eff}} = (1.20)^1 - 1 \]

\[ i_{\text{eff}} = 0.20 = 20\% \]

**Semiannual**

\[ i_{\text{eff}} = (1 + 0.20/2)^2 - 1 \]

\[ i_{\text{eff}} = (1.10)^2 - 1 \]

\[ i_{\text{eff}} = 0.210 = 21\% \]

**Quarterly**

\[ i_{\text{eff}} = (1 + 0.20/4)^4 - 1 \]

\[ i_{\text{eff}} = (1.05)^4 - 1 \]

\[ i_{\text{eff}} = 0.216 = 21.6\% \]
Chapter 4  Time Value of Money  91

P4-35. LG 5: Compounding Frequency, Time Value, and Effective Annual Rates
Intermediate

(a) Compounding Frequency: \( FV_n = PV \times FVIF_{i%,n} \)

\[ \begin{align*}
A \quad & FV_5 = 2,500 \times (FVIF_{3\%,10}) \\
& FV_5 = 2,500 \times (1.344) \\
& FV_5 = 3,360 \\
& \text{Calculator solution:} \ 3,359.79
\end{align*} \]

\[ \begin{align*}
B \quad & FV_3 = 50,000 \times (FVIF_{2\%,18}) \\
& FV_3 = 50,000 \times (1.428) \\
& FV_3 = 71,400 \\
& \text{Calculator solution:} \ 71,412.31
\end{align*} \]

\[ \begin{align*}
C \quad & FV_{10} = 1,000 \times (FVIF_{5\%,10}) \\
& FV_{10} = 1,000 \times (1.629) \\
& FV_{10} = 1,629 \ \\
& \text{Calculator solution:} \ 1,628.89
\end{align*} \]

\[ \begin{align*}
D \quad & FV_6 = 20,000 \times (FVIF_{4\%,24}) \\
& FV_6 = 20,000 \times (2.563) \\
& FV_6 = 51,260 \\
& \text{Calculator solution:} \ 51,266.08
\end{align*} \]

(b) Effective Interest Rate: \( i_{\text{eff}} = (1 + \frac{i}{m})^m - 1 \)

\[ \begin{align*}
A \quad & i_{\text{eff}} = (1 + 0.06/2)^2 - 1 \\
& i_{\text{eff}} = (1 + 0.03)^2 - 1 \\
& i_{\text{eff}} = (1.061) - 1 \\
& i_{\text{eff}} = 0.061 = 6.1\%
\end{align*} \]

\[ \begin{align*}
B \quad & i_{\text{eff}} = (1 + 0.12/6)^6 - 1 \\
& i_{\text{eff}} = (1 + 0.02)^6 - 1 \\
& i_{\text{eff}} = (1.126) - 1 \\
& i_{\text{eff}} = 0.126 = 12.6\%
\end{align*} \]

\[ \begin{align*}
C \quad & i_{\text{eff}} = (1 + 0.05/1)^1 - 1 \\
& i_{\text{eff}} = (1 + 0.05)^1 - 1 \\
& i_{\text{eff}} = (1.05) - 1 \\
& i_{\text{eff}} = 0.05 = 5\%
\end{align*} \]

\[ \begin{align*}
D \quad & i_{\text{eff}} = (1 + 0.16/4)^4 - 1 \\
& i_{\text{eff}} = (1 + 0.04)^4 - 1 \\
& i_{\text{eff}} = (1.170) - 1 \\
& i_{\text{eff}} = 0.17 = 17\%
\end{align*} \]

(c) The effective rates of interest rise relative to the stated nominal rate with increasing compounding frequency.

P4-36. LG 5: Continuous Compounding: \( FV_{\text{cont.}} = PV \times e^{x} \ (e = 2.7183) \)
Intermediate

\[ \begin{align*}
A \quad & FV_{\text{cont.}} = 1,000 \times e^{0.18} = 1,197.22 \\
B \quad & FV_{\text{cont.}} = 600 \times e^{1} = 1,630.97 \\
C \quad & FV_{\text{cont.}} = 4,000 \times e^{0.56} = 7,002.69 \\
D \quad & FV_{\text{cont.}} = 2,500 \times e^{0.48} = 4,040.19 \\
\end{align*} \]

Note: If calculator doesn’t have \( e^{x} \) key, use \( y^x \) key, substituting 2.7183 for \( y \).

P4-37. LG 5: Compounding Frequency and Time Value Challenge
Challenge

(a) (1) \( FV_{10} = 2,000 \times (FVIF_{8\%,10}) \)

\[ \begin{align*}
& FV_{10} = 2,000 \times (2.159) \\
& FV_{10} = 4,318 \\
& \text{Calculator solution:} \ 4,317.85
\end{align*} \]

(2) \( FV_{10} = 2,000 \times (FVIF_{4\%,20}) \)

\[ \begin{align*}
& FV_{10} = 2,000 \times (2.191) \\
& FV_{10} = 4,382 \\
& \text{Calculator solution:} \ 4,382.25
\end{align*} \]

(b) \( FV_{10} = 2,000 \times (FVIF_{0.022\%,3650}) \)

\[ \begin{align*}
& FV_{10} = 2,000 \times (2.232) \\
& FV_{10} = 4,464 \\
& \text{Calculator solution:} \ 4,450.69
\end{align*} \]

(4) \( FV_{10} = 2,000 \times (e^{0.8}) \)

\[ \begin{align*}
& FV_{10} = 2,000 \times (2.226) \\
& FV_{10} = 4,452 \\
& \text{Calculator solution:} \ 4,451.08
\end{align*} \]
(b) (1) \( i_{\text{eff}} = (1 + 0.08/1)^1 - 1 \)  
\( i_{\text{eff}} = (1 + 0.08)^1 - 1 \)  
\( i_{\text{eff}} = (1.08) - 1 \)  
\( i_{\text{eff}} = 0.08 = 8\% \)

(2) \( i_{\text{eff}} = (1 + 0.08/2)^2 - 1 \)  
\( i_{\text{eff}} = (1 + 0.04)^2 - 1 \)  
\( i_{\text{eff}} = (1.082) - 1 \)  
\( i_{\text{eff}} = 0.082 = 8.2\% \)

(3) \( i_{\text{eff}} = (1 + 0.08/365)^{365} - 1 \)  
\( i_{\text{eff}} = (1 + 0.00022)^{365} - 1 \)  
\( i_{\text{eff}} = (1.0833) - 1 \)  
\( i_{\text{eff}} = 0.0833 = 8.33\% \)

(4) \( i_{\text{eff}} = (e^{k} - 1) \)  
\( i_{\text{eff}} = (e^{0.08} - 1) \)  
\( i_{\text{eff}} = (1.0833 - 1) \)  
\( i_{\text{eff}} = 0.0833 = 8.33\% \)

(c) Compounding continuously will result in $134 more dollars at the end of the 10 year period than compounding annually.

(d) The more frequent the compounding the larger the future value. This result is shown in part a by the fact that the future value becomes larger as the compounding period moves from annually to continuously. Since the future value is larger for a given fixed amount invested, the effective return also increases directly with the frequency of compounding. In part b we see this fact as the effective rate moved from 8% to 8.33% as compounding frequency moved from annually to continuously.

P4-38. LG 5: Comparing Compounding Periods

Challenge

(a) \( FV_n = PV \times FVIF_{i\%,n} \)

(1) **Annually:** \( FV = PV \times FVIF_{12\%,2} = $15,000 \times (1.254) = $18,810 \)  
Calculator solution: $18,816

(2) **Quarterly:** \( FV = PV \times FVIF_{3\%,8} = $15,000 \times (1.267) = $19,005 \)  
Calculator solution: $19,001.55

(3) **Monthly:** \( FV = PV \times FVIF_{1\%,24} = $15,000 \times (1.270) = $19,050 \)  
Calculator solution: $19,046.02

(4) **Continuously:** \( FV_{\text{cont.}} = PV \times e^{xt} \)  
\( FV = PV \times 2.7183^{0.24} = $15,000 \times 1.27125 = $19,068.77 \)  
Calculator solution: $19,068.74

(b) The future value of the deposit increases from $18,810 with annual compounding to $19,068.77 with continuous compounding, demonstrating that future value increases as compounding frequency increases.

(c) The maximum future value for this deposit is $19,068.77, resulting from continuous compounding, which assumes compounding at every possible interval.

P4-39. LG 3, 5: Annuities and Compounding: \( FVA_n = PMT \times (FVIFA_{i\%,n}) \)

Intermediate

(a)

(1) **Annual**
\( FVA_{10} = $300 \times (FVIFA_{8\%,10}) \)  
\( FVA_{10} = $300 \times (14.487) \)  
\( FVA_{10} = $4,346.10 \)  
Calculator solution: $4,345.97

(2) **Semiannual**
\( FVA_{10} = $150 \times (FVIFA_{4\%,20}) \)  
\( FVA_{10} = $150 \times (29.778) \)  
\( FVA_{10} = $4,466.70 \)  
Calculator Solution: $4,466.71
(3) **Quarterly**

\[
\begin{align*}
FVA_{10} &= 75 \times (FVIFA_{2\%,40}) \\
FVA_{10} &= 75 \times (60.402) \\
FVA_{10} &= \$4,530.15 \\
\text{Calculator solution: } &\$4,530.15
\end{align*}
\]

(b) The sooner a deposit is made the sooner the funds will be available to earn interest and contribute to compounding. Thus, the sooner the deposit and the more frequent the compounding, the larger the future sum will be.

**P4-40. LG 6: Deposits to Accumulate Growing Future Sum**

\[
PMT = \frac{FVA_n}{FVIFA_{i\%,n}}
\]

<table>
<thead>
<tr>
<th>Case</th>
<th>Terms</th>
<th>Calculation</th>
<th>Payment</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12%, 3 yrs.</td>
<td>PMT = $5,000 ÷ 3.374</td>
<td>$1,481.92</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Calculator solution: $1,481.74</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>7%, 20 yrs.</td>
<td>PMT = $100,000 ÷ 40.995</td>
<td>$2,439.32</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Calculator solution: $2,439.29</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>10%, 8 yrs.</td>
<td>PMT = $30,000 ÷ 11.436</td>
<td>$2,623.29</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Calculator solution: $2,623.32</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>8%, 12 yrs.</td>
<td>PMT = $15,000 ÷ 18.977</td>
<td>$790.43</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Calculator solution: $790.43</td>
<td></td>
</tr>
</tbody>
</table>

**P4-41. LG 6: Creating a Retirement Fund**

Intermediate

(a) \[ PMT = FVA_{42} ÷ (FVIFA_{8\%,42}) \]
\[ PMT = \$220,000 ÷ (304.244) \]
\[ PMT = \$723.10 \]

(b) \[ FVA_{42} = PMT \times (FVIFA_{8\%,42}) \]
\[ FVA_{42} = \$600 \times (304.244) \]
\[ FVA_{42} = \$182,546.40 \]

**P4-42. LG 6: Accumulating a Growing Future Sum**

Intermediate

\[ FV_n = PV \times (FVIF_{i\%,n}) \]
\[ FV_{20} = \$185,000 \times (FVIF_{6\%,20}) \]
\[ FV_{20} = \$185,000 \times (3.207) \]
\[ FV_{20} = \$593,295 = \text{Future value of retirement home in 20 years.} \]
\[ \text{Calculator solution: } \$593,320.06 \]

\[ PMT = FV ÷ (FVIFA_{i\%,n}) \]
\[ PMT = \$272,595 ÷ (FVIFA_{10\%,20}) \]
\[ PMT = \$272,595 ÷ (57.274) \]
\[ PMT = \$4,759.49 \]
\[ \text{Calculator solution: } \$4,759.61 = \text{annual payment required.} \]
P4-43. LG 3, 6: Deposits to Create a Perpetuity  
Intermediate  
(a) Present value of a perpetuity = PMT \times (1 \div i)  
= $6,000 \times (1 \div 0.10)  
= $6,000 \times 10  
= $60,000  
(b) PMT = FVA \div (FVIFA_{10\%,10})  
PMT = $60,000 \div (15.937)  
PMT = $3,764.82  
Calculator solution: $3,764.72

P4-44. LG 2, 3, 6: Inflation, Time Value, and Annual Deposits  
Challenge  
(a) \( FV_n = PV \times (FVIF_{5\%,25}) \)  
\( FV_{20} = $200,000 \times (FVIF_{5\%,25}) \)  
\( FV_{20} = $200,000 \times (3.386) \)  
\( FV_{20} = $677,200 = \) Future value of retirement home in 25 years.  
Calculator solution: $677,270.99  
(b) PMT = \( FV \div (FVIFA_{9\%,25}) \)  
PMT = $677,200 \div (84.699)  
PMT = $7,995.37  
Calculator solution: $7,995.19 = \) annual payment required.  
(c) Since John will have an additional year on which to earn interest at the end of the 25 years his annuity deposit will be smaller each year. To determine the annuity amount John will first discount back the $677,200 one period.  
\( PV_{24} = $677,200 \times 0.9174 = $621,263.28 \)  
John can solve for his annuity amount using the same calculation as in part (b).  
PMT = \( FV \div (FVIFA_{9\%,25}) \)  
PMT = $621,263.28 \div (84.699)  
PMT = $7,334.95  
Calculator solution: $7,334.78 = \) annual payment required.
P4-45.  LG 6: Loan Payment: \[ \text{PMT} = \frac{\text{PVA}}{\text{PVIFA}_{i\%, n}} \]

**Basic Loan**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>PMT = $12,000 \div (\text{PVIFA}_{8%, 3})</td>
<td>B</td>
</tr>
<tr>
<td></td>
<td>PMT = $12,000 \div 2.577</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PMT = $4,656.58</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Calculator solution: $4,656.40</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>PMT = $75,000 \div (\text{PVIFA}_{10%, 30})</td>
<td>D</td>
</tr>
<tr>
<td></td>
<td>PMT = $75,000 \div 9.427</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PMT = $7,955.87</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Calculator solution: $7,955.94</td>
<td></td>
</tr>
</tbody>
</table>

P4-46.  LG 6: Loan Amortization Schedule

**Intermediate**

(a) \[ \text{PMT} = $15,000 \div (\text{PVIFA}_{14\%, 3}) \]

<table>
<thead>
<tr>
<th></th>
<th>Loan Payment</th>
<th>Beginning of Year Principal</th>
<th>Payments</th>
<th>End of Year Principal</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$6,459.95</td>
<td>$15,000.00</td>
<td>$2,100.00 $4,359.95</td>
<td>$10,640.05</td>
</tr>
<tr>
<td>2</td>
<td>$6,459.95</td>
<td>10,640.05</td>
<td>1,489.61 4,970.34</td>
<td>5,669.71</td>
</tr>
<tr>
<td>3</td>
<td>$6,459.95</td>
<td>5,669.71</td>
<td>793.76 5,666.19</td>
<td>0</td>
</tr>
</tbody>
</table>

(The difference in the last year’s beginning and ending principal is due to rounding.)

(c) Through annual end-of-the-year payments, the principal balance of the loan is declining, causing less interest to be accrued on the balance.
P4-47. LG 6: Loan Interest Deductions

Challenge

(a) \[ PMT = \frac{\$10,000}{PVIFA_{13\%},3} \]
\[ PMT = \frac{\$10,000}{2.361} \]
\[ PMT = \$4,235.49 \]
Calculator solution: $4,235.22

(b)

<table>
<thead>
<tr>
<th>End of Year</th>
<th>Loan Payment</th>
<th>Beginning of Year Principal</th>
<th>Payments</th>
<th>End of Year Principal</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$4,235.49</td>
<td>$10,000.00</td>
<td>$1,300.00</td>
<td>$2,935.49</td>
</tr>
<tr>
<td>2</td>
<td>4,235.49</td>
<td>7,064.51</td>
<td>918.39</td>
<td>3,317.10</td>
</tr>
<tr>
<td>3</td>
<td>4,235.49</td>
<td>3,747.41</td>
<td>487.16</td>
<td>3,748.33</td>
</tr>
</tbody>
</table>

(The difference in the last year’s beginning and ending principal is due to rounding.)

P4-48. LG 6: Monthly Loan Payments

Challenge

(a) \[ PMT = \frac{\$4,000}{PVIFA_{1\%},24} \]
\[ PMT = \frac{\$4,000}{21.243} \]
\[ PMT = \$188.28 \]
Calculator solution: $188.29

(b) \[ PMT = \frac{\$4,000}{PVIFA_{0.75\%},24} \]
\[ PMT = \frac{\$4,000}{21.889} \]
\[ PMT = \$182.74 \]
Calculator solution: $182.74

P4-49. LG 6: Growth Rates

Basic

(a) \[ PV = FV_n \times PVIF_{i\%},n \]

Case

A \[ PV = FV_4 \times PVIF_{k\%},4yrs. \]
\$500 = $800 \times PVIF_{k\%},4yrs
0.625 = PVIF_{k\%},4yrs
12\% < k < 13\%
Calculator Solution: 12.47%

B \[ PV = FV_9 \times PVIF_{k\%},9yrs. \]
\$1,500 = $2,280 \times PVIF_{k\%},9yrs
0.658 = PVIF_{k\%},9yrs.
4\%<k<5\%
Calculator solution: 4.76%

C \[ PV = FV_6 \times PVIF_{k\%},6 \]
\$2,500 = $2,900 \times PVIF_{k\%},6yrs.
0.862 = PVIF_{k\%},6yrs.
2\% < k < 3\%
Calculator solution: 2.50%
(b) Case
A Same as in (a)
B Same as in (a)
C Same as in (a)

c) The growth rate and the interest rate should be equal, since they represent the same thing.

P4-50. LG 6: Rate of Return: \( PV_n = FV_n \times (PVIF_{\%i,n}) \)
Intermediate

(a) \( PV = \$2,000 \times (PVIF_{\%i,3\text{yrs.}}) \)
\$1,500 = \$2,000 \times (PVIF_{\%i,3\text{yrs.}}) 
0.75 = PVIF_{\%i,3\text{yrs.}} 
10\% < i < 11\%
Calculator solution: 10.06%

(b) Mr. Singh should accept the investment that will return \$2,000 because it has a higher return for the same amount of risk.

P4-51. LG 6: Rate of Return and Investment Choice
Intermediate

(a) A \( PV = \$8,400 \times (PVIF_{\%i,6\text{yrs.}}) \)
\$5,000 = \$8,400 \times (PVIF_{\%i,6\text{yrs.}}) 
0.595 = PVIF_{\%i,6\text{yrs.}} 
9\% < i < 10\%
Calculator solution: 9.03%

B \( PV = \$15,900 \times (PVIF_{\%i,15\text{yrs.}}) \)
\$5,000 = \$15,900 \times (PVIF_{\%i,15\text{yrs.}}) 
0.314 = PVIF_{\%i,15\text{yrs.}} 
8\% < i < 9\%
Calculator solution: 8.02%

(b) Investment C provides the highest return of the 4 alternatives. Assuming equal risk for the alternatives, Clare should choose C.

P4-52. LG 6: Rate of Return- Annuity: \( PVA_n = PMT \times (PVIFA_{\%i,n}) \)
Basic
\$10,606 = \$2,000 \times (PVIFA_{\%i,10\text{yrs.}}) 
5.303 = PVIFA_{\%i,10\text{yrs.}} 
13\% < i < 14\%
Calculator solution: 13.58%
P4-53. LG 6: Choosing the Best Annuity: \( PVA_n = PMT \times (PVIFA_{i,n}) \)

Intermediate

(a) **Annuity A**
\[
30,000 = 3,100 \times (PVIFA_{i,20\text{ yrs.}}) \\
9.677 = PVIFA_{i,20\text{ yrs.}} \\
8\% < i < 9\% \\
\text{Calculator solution: 8.19%}
\]

**Annuity B**
\[
25,000 = 3,900 \times (PVIFA_{i,10\text{ yrs.}}) \\
6.410 = PVIFA_{i,10\text{ yrs.}} \\
9\% < i < 10\% \\
\text{Calculator solution: 9.03%}
\]

(b) Loan B gives the highest rate of return at 9% and would be the one selected based upon Raina’s criteria.

P4-54. LG 6: Interest Rate for an Annuity

Challenge

(a) **Defendants interest rate assumption**
\[
2,000,000 = 156,000 \times (PVIFA_{i,25\text{ yrs.}}) \\
12.821 = PVIFA_{i,25\text{ yrs.}} \\
5\% < i < 6\% \\
\text{Calculator solution: 5.97%}
\]

(b) **Prosecution interest rate assumption**
\[
2,000,000 = 255,000 \times (PVIFA_{i,25\text{ yrs.}}) \\
7.843 = PVIFA_{i,25\text{ yrs.}} \\
i = 12\% \\
\text{Calculator solution: 12.0%}
\]

(c) \[
2,000,000 = PMT \times (PVIFA_{i,25\text{ yrs.}}) \\
2,000,000 = PMT (9.823) \\
PMT = 203,603.79
\]
P4-55. LG 6: Loan Rates of Interest: \( PVA_n = PMT \times (PVIFA_{i%,n}) \)

Intermediate

(a) \textbf{Loan A}

\[
\begin{align*}
\text{Loan C} & \\
$5,000 &= $2,010.45 \times (PVIFA_{i%,3\ \text{yrs.}}) \\
i &= 10\%
\end{align*}
\]

(b) Mr. Fleming should choose Loan B, which has the lowest interest rate.

P4-56. LG 6: Number of Years to Equal Future Amount

Intermediate

(a) \textbf{Loan B}

\[
\begin{align*}
\text{Loan B} & \\
$5,000 &= $1,543.21 \times (PVIFA_{i%,4\ \text{yrs.}}) \\
i &= 9\%
\end{align*}
\]

P4-57. LG 6: Time to Accumulate a Given Sum

Intermediate

(a) \textbf{Loan B}

\[
\begin{align*}
\text{Loan B} & \\
$5,000 &= $1,543.21 \times (PVIFA_{i%,4\ \text{yrs.}}) \\
i &= 9\%
\end{align*}
\]

(b) Mr. Fleming should choose Loan B, which has the lowest interest rate.

P4-56. LG 6: Number of Years to Equal Future Amount

Intermediate

(a) \textbf{Loan A}

\[
\begin{align*}
\text{Loan A} & \\
$5,000 &= $1,352.81 \times (PVIFA_{i%,5\ \text{yrs.}}) \\
i &= 11\%
\end{align*}
\]

(b) Mr. Fleming should choose Loan B, which has the lowest interest rate.

P4-56. LG 6: Number of Years to Equal Future Amount

Intermediate

(a) \textbf{Loan B}

\[
\begin{align*}
\text{Loan B} & \\
$5,000 &= $1,543.21 \times (PVIFA_{i%,4\ \text{yrs.}}) \\
i &= 9\%
\end{align*}
\]

(b) Mr. Fleming should choose Loan B, which has the lowest interest rate.

P4-57. LG 6: Time to Accumulate a Given Sum

Intermediate

(a) \textbf{Loan B}

\[
\begin{align*}
\text{Loan B} & \\
$5,000 &= $1,543.21 \times (PVIFA_{i%,4\ \text{yrs.}}) \\
i &= 9\%
\end{align*}
\]

(b) Mr. Fleming should choose Loan B, which has the lowest interest rate.

P4-56. LG 6: Number of Years to Equal Future Amount

Intermediate

(a) \textbf{Loan A}

\[
\begin{align*}
\text{Loan A} & \\
$5,000 &= $1,352.81 \times (PVIFA_{i%,5\ \text{yrs.}}) \\
i &= 11\%
\end{align*}
\]

(b) Mr. Fleming should choose Loan B, which has the lowest interest rate.
P4-58. LG 6: Number of Years to Provide a Given Return
Intermediate

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>PVA = PMT × (PVIFA_{11%,n yrs.})</td>
<td>B</td>
</tr>
<tr>
<td></td>
<td>$1,000 = $250 × (PVIFA_{11%,n yrs.})</td>
<td>$150,000 = $30,000 × (PVIFA_{15%,n yrs.})</td>
</tr>
<tr>
<td></td>
<td>4.000 = PVIFA_{11%,n yrs.}</td>
<td>5.000 = PVIFA_{15%,n yrs.}</td>
</tr>
<tr>
<td></td>
<td>5 &lt; n &lt; 6</td>
<td>9 &lt; n &lt; 10</td>
</tr>
<tr>
<td></td>
<td>Calculator solution: 5.56</td>
<td>Calculator solution: 9.92</td>
</tr>
</tbody>
</table>

C PVA = PMT × (PVIFA_{10\%,n yrs.})

$80,000 = $30,000 × (PVIFA_{10\%,n yrs.})

2.667 = PVIFA_{10\%,n yrs.}

3 < n < 4

Calculator solution: 3.25

E PVA = PMT × (PVIFA_{6\%,n yrs.})

$17,000 = $3,500 × (PVIFA_{6\%,n yrs.})

4.857 = PVIFA_{6\%,n yrs.}

5 < n < 6

Calculator solution: 5.91

P4-59. LG 6: Time to Repay Installment Loan
Intermediate

<p>| | | |</p>
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(a) $14,000 = $2,450 × (PVIFA_{12%,n yrs.})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.714 = PVIFA_{12%,n yrs.}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10 &lt; n &lt; 11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Calculator solution: 10.21</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(b) $14,000 = $2,450 × (PVIFA_{9%,n yrs.})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.714 = PVIFA_{9%,n yrs.}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8 &lt; n &lt; 9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Calculator solution: 8.37</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(c) $14,000 = $2,450 × (PVIFA_{15%,n yrs.})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.714 = PVIFA_{15%,n yrs.}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13 &lt; n &lt; 14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Calculator solution: 13.92</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(d) The higher the interest rate the greater the number of time periods needed to repay the loan fully.</td>
</tr>
</tbody>
</table>
P4-60. Ethics Problem
Intermediate

This is a tough issue. Even back in the Middle Ages, scholars debated the idea of a “just price.” The ethical debate hinges on (1) the basis for usury laws, (2) whether full disclosure is made of the true cost of the advance, and (3) whether customers understand the disclosures. Usury laws are premised on the notion that there is such a thing as an interest rate (price of credit) that is “too high.” A centuries-old fairness notion guides us into not taking advantage of someone in duress or facing an emergency situation. One must ask, too, why there are not market-supplied credit sources for borrowers, which would charge lower interest rates and receive an acceptable risk-adjusted return. On issues #2 and #3, there is no assurance that borrowers comprehend or are given adequate disclosures. See the box for the key ethics issues on which to refocus attention (some would view the objection cited as a smokescreen to take our attention off the true ethical issues in this credit offer).