Moral Hazard

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Hidden Information: Agent has type or information that principal does not know. (Common values: Agent’s type affects both players; private values: agent’s type affects only agent.

Hidden action/Moral Hazard: Agent takes an action that is unobservable to principal but affects his payoff. There is signal of the agent’s action (e.g. output) that is verifiable/contractible and can be used to give incentives.

Themes: Basic trade off between risk-sharing versus incentives; “sell firm to agent”; giving agent a surplus, an efficiency wage, to work; use all informative signals to design incentives but not more; free-rider problem in teams; relative performance evaluation.
Basic Idea

The employer, or “The Principal”, hires a worker (“The Agent”). The result of the agent’s work could be “Good” or “Bad”. Hard work (known as “High Effort”) increases the chance of success, but does not guarantee it. Even though the result of the agent’s work is correlated with the agent’s effort, it is not possible to tell how hard he worked. We assume the principal is risk neutral and agent is risk averse. The principal offers the agent a contract, recognizing that the terms of the contract will determine the incentives of the agent.

Problem:
A. If the agent’s wage varies with the result of the work THEN....

B. If the agent’s wage doesn’t vary with the result of the work THEN....

Implication:
An agent can work, \( a = 1 \), or shirk, \( a = 0 \). The cost of shirking is normalized to zero and the cost of working is \( \psi > 0 \). The agent’s payoff if he gets wage \( w \) is

\[
U(w) - \psi a
\]

where \( U \) is strictly increasing \( U' > 0 \) and concave \( U'' \leq 0 \).

The principal cannot observe the action the agent takes but can observe whether output is high \( x_1 \) or low \( x_0 \). The probability of high output is \( p_a \) and we assume \( p_0 < p_1 \). We assume the agent’s output is observable and verifiable so the agent’s wage can be made contingent on it.

Not only is output a signal of the agent’s actions but it is also enters the principal’s payoff:

\[
V(x_i - w_i)
\]

\( V' > 0 \), \( V'' \leq 0 \) \( i \in \{0, 1\} \).
First-best:

Suppose $a$ is verifiable so there is no moral hazard. Principal sets $a, w_1, w_0$ to maximize

$$\max_{a, w_1, w_0} p_a V(x_1 - w_1) + (1 - p_a) V(x_0 - w_0)$$

s.t. $p_a u(w_1) + (1 - p_a) u(w_0) - \psi a \geq U(IR)$

where $U$ is the agent’s outside option and this constraint is called the individual rationality constraint or participation constraint.

Step 1: The participation constraint must bind (i.e. hold as an equality) at the optimum: otherwise, lower $w_0$ say, increase principal’s maximand without violating $IR$. Let $\lambda$ be the Lagrangean multiplier for the constraint.

Step 2: Take first-order conditions w.r.t. $w_1$ and $w_0$:

$$\frac{V'(x_1 - w_1)}{u'(w_1)} = \frac{V'(x_0 - w_0)}{u'(w_0)} = \lambda$$

Borch rule.
**Observation:** Assume at least one of the utility functions is strictly concave. The Borch rule implies *monotonicity*: wages increases with output $w_1 \geq w_0$. Suppose not and $w_1 < w_0$. Then, as $x_1 > x_0$:

$$
\frac{V'(x_1 - w_1)}{u'(w_1)} < \frac{V'(x_0 - w_0)}{u'(w_0)}
$$

contradicting the Borch rule. Monotonicity is an implication of efficient insurance and has nothing to do with incentives as effort is verifiable. But it is also natural to expect monotonic payments to be important when incentives are an issue. We will find conditions that imply monotonicity at the second-best and study in general how incentive provision alters the Borch rule.
Example: Risk-neutral principal, risk-averse agent

The Borch rule reduces to

\[
\frac{1}{u'(w_1)} = \frac{1}{u'(w_0)}
\]

so we have \( w_1 = w_0 = w^* \) such that

\[
u(w^*) - \psi a^* = U
\]

where \( a^* \) is the optimal effort for the principal to induce. The principal fully insures the agent. If the principal introduced wage variation while keeping the expected wage constant, the agent’s expected utility falls and he refuses to participate. The principal would have to increase expected wages to persuade agent to participate and induce the same effort choice. It is better to eliminate wage variation and decrease the expected wage.
Example: Risk-neutral agent

Then Borch rule implies

\[ x_1 - w_1 = x_0 - w_0 = \alpha^* \]

so

\[ w_i = x_i - \alpha^*. \]

Then, as \( V \) is monotone principal’s problem becomes

\[
\max_{a} \alpha^* \\
\text{s.t. } p_a x_1 + (1 - p_a) x_0 - \alpha^* - \psi a = U.
\]

or

\[
\max_a p_a x_1 + (1 - p_a) x_0 - U - \psi a.
\]

The interpretation is that the principal “sells the firm” to the agent for \( \alpha^* \) and the agent chooses effort to maximize expected profits net of cost of effort. The principal takes the effort the agent chooses and his outside option into account when he sets \( \alpha^* \).
Now effort is unobservable to principal. We begin with a basic point: the principal’s welfare is (weakly) lower at the second-best than the first-best. The principal can always replicate the second-best when effort is observable by offering the second-best contract and leaving the choice of effort up to the agent. We assume for simplicity that principal is risk-neutral. We determine principal’s expected payoff from achieving each effort level and then optimize among effort levels.
First consider the case where principal wants to the agent to set $a = 1$. To make the agent work rather than shirk it must be the case that working is *incentive compatible*

\[ p_1 u(w_1) + (1 - p_1) u(w_0) - \psi \geq p_0 u(w_1) + (1 - p_0) u(w_0) \quad (IC) \]

or, rearranging:

\[ (p_1 - p_0) (u(w_1) - u(w_0)) \geq \psi \]

Therefore, it must be the case that $w_1 > w_0$ so the wage function is *monotonic* in output. Also, to get the agent to participate in the first place, he must be paid at least his outside option $U$ so his decision is *individually rational*.

\[ p_1 u(w_1) + (1 - p_1) u(w_0) - \psi \geq U. \]
Proofs:
Step 1: The IR constraint must *bind* (i.e. hold as an equality) at the optimum:

Step 2: *IC* must bind at the optimum:

Essentially, if *IC* does not bind, the principal is offering the agent more insurance as the agent is risk averse while the principal is risk neutral. The principal can therefore offer the agent a lower *expected* wage and increase his own profits.

Let $W^1$ be the principal’s welfare from getting the agent to work. If he lets the agent shirk, he pays him a constant wage $w^*$ so $u(w^*) = U$. Let $W^0$ be the principal’s payoff in this case. Therefore, the principal gives the agent the incentive to work rather than shirk if and only if

$$W^1 \geq W^0.$$
Our conclusions are that if the principal wants the agent to work:
(1) The IR constraint binds;
(2) The IC constraint binds;
(3) The agent gets paid more for higher outputs - wages are monotonic in output.
We will see when we look at more general models that (1) and (2) still hold in some sense but (3) does not without more assumptions.
Organizational Change at Safelite Glass Corporation

• Largest US installer of automobile glass.
• During 1994/95 they moved from hourly wages to piece rate pay.
• A study tracked 3,000 employees over 19 months as the piece rates were gradually phased in.

• What happened?

Source: E. Lazear 2000
Organizational Change at Safelite Glass Corporation

Safelite Productivity Before & After Introduction of Piece-Rates

Source: E.Lazear 2000
Organizational Change at Safelite Glass Corporation

- **Variance** in output across workers increased significantly.
- **Customer satisfaction** index increased from 90% to 94%.
- **Sick leave fell** dramatically: mean level of paid sick hours fell by almost 60%.
- Overall **pay per incumbent worker increased** 10%. 90% of workers had higher pay in the piece-rate system.
- **Overall profits went up.**

Source: E.Lazear 2000
Big Picture: Is it possible to explain involuntary unemployment using incentive theory? i.e. there is excess supply of labour but wages do not go down to market-clearing level.
Suppose principal and agent are risk-neutral but wages must be non-negative as agent has no wealth (more broadly there can be an upper bound on transfers from agent to principal): \( w_i \geq 0 \).

Suppose the agent’s outside option is also zero.

We know that with unlimited liability the principal can achieve the first-best.

Suppose \( a = 1 \) at first-best and how consider implementing this effort with limited liability.
Step 1. The $IC$ constraint becomes:

Step 2. Notice the $RHS$ is non-negative as there is limited liability and hence the $IC$ constraint implies the $IR$ constraint is automatically satisfied:

Hence, we can drop it from the program.

Step 3. Next, at the optimum we must have $w_0 = 0$.

Step 4. The $IC$ constraint must bind at the optimum (i.e. hold as an equality).
We conclude

\[ p_1 w_1 - \psi = p_0 w_1 \text{ so} \]

\[ w_1 = \frac{\psi}{p_1 - p_0} \]

Hence, RHS of IC is strictly positive. The agent earns rents or an efficiency wage in equilibrium. The principal’s payoff is then

\[ p_1 x_1 + (1 - p_1) x_0 - \frac{p_1 \psi}{p_1 - p_0} < p_1 x_1 + (1 - p_1) x_0 - \psi. \]

In fact, if the efficiency wage is high the principal may find it optimal to implement \( a = 0 \) under limited liability when \( a = 1 \) is first-best effort level with unlimited liability.
Two issues: the free-rider problem in teams and relative performance evaluation.
There are $N$ agents all taking action $a_i \in A_i = [a, \bar{a}]$.

The total output $x : A \rightarrow \mathbb{R}$ of the agents is a deterministic function of the profile of actions/efforts of the agents $a \in A \equiv A_1 \times \cdots \times A_N$. We assume $x$ is increasing, concave and differentiable.

Individual effort and output is unobservable.

We assume each agent is risk-neutral and had a payoff function

$$w_i - \psi_i(a_i)$$

where $\psi_i$ is increasing, differentiable and strictly convex.
A contract \( w = (w_1, \ldots, w_N) \) specifies wages as a function of output \( x \) (these are sometimes called sharing rules). Holmström (1982) begins by assuming that the \( N \) agents are a partnership and cannot dispose of any money so

\[
\sum_{i=1}^{N} w_i(x) = x \quad \forall x.
\]  

and budget balance has to be satisfied. The first best effort level \( a^* \) is given by

\[
a^* = \arg \max_{a \in A} x(a) - \sum_{i=1}^{N} \psi_i(a_i).
\]
Since, the actions are unobservable, we ask whether there is a contract $w$ with a non-cooperative Nash equilibrium which implements the first-best action profile. If the sharing rules are differentiable, we find that at a Nash equilibrium

$$w_i' x_i' - \psi_i' = 0 \ \forall i = 1, \ldots, N \tag{2}$$

where $x_i' = \frac{\partial x}{\partial a_i}$. At the first best, we have

$$x_i' - \psi_i' = 0 \ \forall i = 1, \ldots, N. \tag{3}$$

For the first best to be implementable, by (2) and (3), we must have

$$w_i' = 1 \ \forall i = 1, \ldots, N.$$

But this contradicts (1) as differentiating it implies

$$\sum_{i=1}^{N} w_i' = 1.$$
In general, we have

**Theorem**

Suppose \( a^* \in \text{int}A \) (i.e. all agents provide some effort at first-best effort level). Then, there do not exist wage schedules that satisfy budget balance and yield \( a^* \) as a Nash equilibrium.

The intuition is that each agent’s effort exerts a positive externality on others and therefore each undersupplies effort.
If we can break budget-balance, we can restore efficiency: Let \( w_i(x) \)

\[
  w_i(x) = \begin{cases} 
    b_i & \text{if } x \geq x(a^*) \\
    0 & \text{if } x < x(a^*) 
  \end{cases}
\]

where \( \sum_i b_i = x(a^*) \) and \( b_i > \psi_i(a_i^*) > 0 \). This is possible as \( x(a^*) - \sum_i \psi_i(a_i^*) \geq 0 \) by Pareto optimality. It is clear that \( a^* \) is a Nash equilibrium. This scheme is not renegotiation-proof: If \( x < x(a^*) \) is observed, the partnership has an incentive to rip up the contract and divide up the money. Introducing a Principal (who does not exert effort) and is the residual claimant of output if it is ever below first-best restores credibility.
There are 2 agents all taking action $a_i \in A_i = [a, \bar{a}]$.

There is a signal of each individual’s output $x_i$ this depends on agent $i$’s effort and some random shock. Agent $i$’s random shock may or may not be correlated with agent $j$’s.

When should agent $i$’s compensation depends on agent $j$’s output as well as his own, $w_i(x_i, x_j)$ and when should it be independent of agent $j$’s, $w_i(x_i)$?
Companies are subject to price regulation. Water company in region A is allowed to charge a price that depends on cost reduction in region B.

Region A and B are chosen to they are ex ante similar in terms of geography and weather patterns.
The Pay to Play Principle

Job/worker characteristics

- Good performance measure.
- Not liquidity constrained.
- Tolerant of financial risk.

The best way to pay

Pay to Play:

1. Set piece rate = gross profit margin (or commission rate = 100% etc.).
2. Charge for the right to work.
The Best Way to Pay

Job/worker characteristics

- Good performance measure.
- Not liquidity constrained.
- Tolerant of financial risk.

The best way to pay?
Pay for Performance and Financial Risk

• Suppose:
  – # of windshields = worker’s choice + random shock

• It is still the case that an increase in the piece rate motivates more effort.

• But now an increase in the piece rate also reallocates risk between the firm and the worker.
But Why Does the Allocation of Risk Matter?

- It does not if the firm and the worker are tolerant of risk, that is, if they are “risk neutral.” But it does matter if firm and/or worker are “risk averse.”

- Most workers are risk averse.
- But large firms are often risk neutral.
The Risk-Incentive Trade-Off

- If the worker is risk averse, an increase in the piece rate increases the worker’s risk costs (for which the firm has to compensate him).
- The firm therefore faces a trade-off between allocating risk and motivating the worker:
What To Do About It

• Set the **piece rate below the gross profit margin**.
• Set a higher piece
  ... the **more productive the worker**.
  ... the **less noisy the performance measure**.
  ... the **less risk averse the worker**.
• Hire less risk averse workers. **Less risk averse workers are cheaper to motivate**. Use self-selection to identify less risk averse workers.
• Reduce the noise in your performance measures. **The less noisy the performance measure, the cheaper it is to motivate workers** (“Informativeness Principle”). Example: CEOs
“Fund managers can be paid in a variety of ways, much like a pro athlete. In baseball, a player’s total earnings can be tied to hitting more home runs or making fewer errors. Fund managers also can be paid either for big bets or for playing it safe. Some are paid more for making money in any environment, but most are paid for beating a fluctuating benchmark, such as the Standard & Poor’s 500-stock index.”

WSJ, April 7, 2003
The Best Way to Pay

Job/worker characteristics

Good performance measure.

Not liquidity constrained.

Tolerant of financial risk.

The best way to pay

1. Set piece rate below gross profit margin.

2. Higher piece rate the more the worker can affect the performance measure etc.

3. Hire employees who are more tolerant of risk.

3. The cleaner the performance measure, the better.
The Best Way to Pay

Job/worker characteristics

- Good performance measure.
- Not liquidity constrained.
- Tolerant of financial risk.

The best way to pay

?
Liquidity Constraints

• Last week: the worker could pay the firm.
• Suppose now: the worker is liquidity constrained and cannot pay the firm.
• Now what is the best way to pay? Why?
• Total value in the vertical chain is still maximized by setting the piece rate equal to the gross profit margin. (“Selling firm to agent”)
• But now the firm’s can no longer do that as worker has no money.
• The firm therefore has to pay an efficiency wage.
• As a result, the firm faces a trade-off between value capture and creation.
What To Do About It?

- Hire less constrained workers. **Less constrained workers are cheaper to motivate.**
- Facilitating financing is a win-win. Examples: GM, franchising.
The Best Way to Pay

Job/worker characteristics

- Good performance measure.
- Not liquidity constrained.
- Tolerant of financial risk.

The best way to pay

1. Have to pay efficiency wage
2. Hire less constrained workers.
3. Support financing by those you do hire.
The Best Way to Pay

Job/worker characteristics

- Good performance measure.
- Not liquidity constrained.
- Tolerant of financial risk.

The best way to pay

?
Poor Performance Measures

- Last week: the firm could measure the worker’s performance perfectly (# of windshields).
- Suppose now: the firm also cares about windshield quality but cannot measure it as well as # of windshields installed.
- Now what is the best way to pay? Why?
- Total value in the vertical chain is no longer maximized by setting the piece rate equal to the gross profit margin.
- This common problem is known as the “Multi-tasking Problem” or “The Folly of Rewarding A While Hoping for B.”
- Examples: teacher pay, Tim Hardaway, cancer lab etc.
What To Do About It?

- Balance incentives. Example: repair on own time, bonus.
- Job design. Example: sales and customer service.
- Firm boundaries. Example: trucking, cabs.
- Reduce incentives and monitor. Example: cancer lab.
The Best Way to Pay

Job/worker characteristics

- Good performance measures (Strike through)
- Not liquidity constrained
- Tolerant of financial risk

The best way to pay

1. Balance incentives.
2. Reduce incentives and monitor perf.
3. Possibly adjust job design.
4. Possibly adjust firm boundaries.