Game Trees of Imperfect Information

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February 12, 2014
Kuhn’s Theorem says:

In a game tree of perfect information

Backward-induction $\rightarrow$ Nash equilibrium

Is the converse true, i.e.,

Nash equilibrium $\rightarrow$ Backward-induction?
Q. Can a NE in a zero-sum game exhibit an empty threat?
Matching Pennies

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Eq. Outcome: (-1, 1)
Information Set:

A number of nodes of the same player

The player must make a choice in ignorance of which node applies

Same choice of moves at all the nodes

A game where all information sets are singletons is a
**Game of Perfect Information**

All other games are
**Games of Imperfect Information**
Nature chooses $L$ or $R$ with prob $\alpha$ and $1-\alpha$.

What matters is information, not time:
For which values of $\alpha$ should the player go L?
Penalty Kick

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S

G

L

R

l

r

.5

1

.9

.7
Equilibrium

\[
\begin{array}{c|cc}
 & 3/7 & 4/7 \\
\hline
2/7 & .5 & 1 \\
5/7 & .9 & .7 \\
\end{array}
\]
Check for Striker:

```
    L
   /\ 1
  /  
R   .9
   
     .7
```

Check for Goalie:

```
    .5
   /\  
  /   
R   .9
   
     .9
   
     .1
   
     .9
```
Should I go L or R?
Simple Poker

$20 in pot.

Two cards in a hat: Hi and Lo.

One card chosen at random.

Player I sees the card. Player II does not.

I can **pass** → II gets pot

  **bet**: Add $10 to pot; then

II can **fold** → I gets pot

  **call**: Add $10 to pot; card checked

    Hi → pot to I

    Lo → pot to II
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Can any strategy be deleted by dominance?

What are the equilibrium strategies?

What are the equilibrium payoffs?

How much is I’s information worth?
Suppose II assumes that I plays according to the equilibrium strategy

I has bet. What is the probability that the card is Hi?