Experiments on cooperation in repeated games

Dal Bó (2005)
Dal Bó and Fréchette (2011)
Fréchette and Yuksel (2013)

Presented by Linh T. Tô

07 April 2014
Introduction

• “The theory of repeated games has been somewhat disappointing. … the theory does not make sharp predictions.” (Fudenberg and Maskin, 1993)

• Cooperation can be supported in infinitely repeated games when players are concerned about future rewards and punishments, but when does cooperation in fact occur?

• Series of papers exploring cooperation in the Prisoner’s Dilemma game in the lab:
  ○ “Cooperation under the Shadow of the Future: Experimental Evidence from Infinitely Repeated Games” (Dal Bó, AER, 2005)
  ○ “The Evolution of Cooperation in Infinitely Repeated Games: Experimental Evidence” (Dal Bó and Fréchette, AER, 2011)
  ○ “Infinitely Repeated Games in the Laboratory: Four Perspectives on Discounting and Random Termination” (Fréchette and Yuksel, WP)
Questions

Dal Bó 2005: Compares finitely repeated games with infinitely repeated games using random termination of the same expected length.

- Is the level of cooperation higher when $\delta$ is higher?
- Is the level of cooperation lower in the first round of finitely repeated games than in the first round of infinitely repeated games of the same expected length?
Questions


• Do subjects learn to defect when it is the only equilibrium action?
• Do subjects learn to cooperate when it is an equilibrium action?
• Do subjects learn to cooperate when it is a risk dominant action?
Questions

Fréchette and Yuksel 2013: Compare three different implementations of infinitely repeated games in the lab.

• Do agents respond to payoff discounting and probabilistic continuation differently in repeated interactions?

• Do different methods of implementation lead to different conclusions with respect to basic comparative statics of the theory?
Table 2—Stage Game Payoffs in Points

<table>
<thead>
<tr>
<th></th>
<th>PD1</th>
<th></th>
<th>PD2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Blue player</td>
<td></td>
<td>Blue player</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>D</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>Red player</td>
<td>65, 65</td>
<td>10,100</td>
<td>Red player</td>
<td>75, 75</td>
</tr>
<tr>
<td></td>
<td>100, 10</td>
<td>35, 35</td>
<td>D</td>
<td>100, 10</td>
</tr>
</tbody>
</table>
Dal Bó — Design

• Earnings: 200 points = $1 with $5 show-up fee.
• Rotation matching: avoid potential interaction and contagion effects between the different repeated games.
  ◦ Red group
  ◦ Blue group
• Dice (Infinite): after each round, a die is rolled to determine whether the match ends.
• Finite: common knowledge length.
Dal Bó — Design

<table>
<thead>
<tr>
<th>Finite</th>
<th>Dice</th>
</tr>
</thead>
<tbody>
<tr>
<td>PD1</td>
<td>$H = 1, 2, 4$</td>
</tr>
<tr>
<td>PD2</td>
<td>$H = 1, 2, 4$</td>
</tr>
</tbody>
</table>

- **Order of treatments:**
  - “Normal”: $\delta = 0, \delta = \frac{1}{2}, \delta = \frac{3}{4}$
  - “UD” (upside-down): $\delta = \frac{3}{4}, \delta = \frac{1}{2}, \delta = 0$

- Dice or Finite, PD1 or PD2, Normal or UD

- $N$ subjects in one session yields $N/6$ matches per subject
Dal Bó — Descriptive Data

Eight sessions with three treatments each:

- **Subjects**: UCLA undergraduates, 30 to 60 per session
- **Average earnings**: $13.03 to $23.09 per session
- **Average number of rounds**:
  - $\delta = \frac{1}{2} \quad \text{—} \quad 1.91$
  - $\delta = \frac{3}{4} \quad \text{—} \quad 3.73$
Learning regarding difficulties of cooperation when $\delta = 0$ or finite.

Averaging matches four through ten — cooperation rate: $\delta = 0$: 9 percent; $\delta = \frac{1}{2}$: 27 percent; $\delta = \frac{3}{4}$: 37 percent
Dal Bó — Round Results

Infinitely repeated games versus finitely repeated games

<table>
<thead>
<tr>
<th>Round</th>
<th>$\delta = 0$</th>
<th>1</th>
<th>$\delta = \frac{1}{2}$</th>
<th>2</th>
<th>$\delta = \frac{3}{4}$</th>
<th>3</th>
<th>4</th>
<th>5–12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dice</td>
<td>9.17</td>
<td></td>
<td>30.93</td>
<td>26.10</td>
<td>46.20</td>
<td>40.76</td>
<td>38.76</td>
<td>34.58</td>
</tr>
<tr>
<td>Finite</td>
<td>10.34</td>
<td></td>
<td>13.31</td>
<td>6.90</td>
<td>34.38</td>
<td>21.55</td>
<td>18.97</td>
<td>10.63</td>
</tr>
</tbody>
</table>

Table: Percentage of Cooperation by Round (Matches 4 to 10)

- One-shot games: similar for $\delta = 0$ or $H = 1$.
- Fourth round: $\delta = \frac{3}{4}$ is significantly greater than $H = 4$.
- Final round of $H = 2$ and $H = 4$: similar to one-shot games.
Dal Bó — Summary

- Subjects cooperate more the greater the probability of future interaction.
- Subjects cooperate less in finitely repeated games than in infinitely repeated games of the same expected length.
Dal Bó and Fréchette — Design

Six treatments, one per session (50 minutes):

- $R = 32$, $R = 40$, and $R = 48$
- $\delta = \frac{1}{2}$, and $\delta = \frac{3}{4}$
Blonksi and Spagnolo (2001): Cooperation is risk dominant if playing Grim Trigger (G) is the best response to the other player’s choosing Grim Trigger (G) or Always Defect (AD) with equal probabilities.
Dal Bó and Fréchette — Descriptive Data

- 18 sessions
- 266 NYU undergraduates: 14.78 per session on average (12 to 20)
- Average earning $25.95 ($16.29 to $42.93)
- $\delta = \frac{1}{2}$: 1.96 round per match on average (up to 9)
- $\delta = \frac{3}{4}$: 4.42 round per match on average (up to 23)
Do subjects learn to defect when it is the only equilibrium action?

Figure: First rounds, not SGPE

Drop to one-shot levels (10%) (Cooper et al. 1996; Dal Bó 2005)
Dal Bó and Fréchette — Learning to Cooperate

Do subjects learn to cooperate when it is an equilibrium action?

Figure: First rounds, SGPE but not RD
Dal Bó and Fréchette — Learning to Cooperate

Do subjects learn to cooperate when it is risk dominant?

Figure: First rounds, RD
As subjects gain experience they may modify their behavior.

Cooperation may not prevail even when it is a possible equilibrium action or under more stringent conditions (risk dominance).

Equilibrium selection theory: Contradicts theories that select inefficient outcome even when players are arbitrarily patient and those selecting efficient outcomes whenever they are a possible equilibrium.
Three implementations:

- Randomly terminated (RT): Fixed known probability $\delta$ that the game continues for an additional round, no discounting.
- Payoff discounting followed by random termination (D + RT): Fixed (known) $\rho$ rounds with discount factor $\delta$ are played before RT with no discounting.
- Block random termination (BRT): Feedback about termination in blocks of $\rho$ rounds, and continuation probability of the block is $\delta^\rho$. 
Fréchette and Yuksel — Design

- \( \delta = \frac{3}{4} \)
- \( \rho = 4 \)

First 12 matches (CC is RD):

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>40, 40</td>
<td>12, 48</td>
</tr>
<tr>
<td>D</td>
<td>48, 12</td>
<td>20, 20</td>
</tr>
</tbody>
</table>

Last 6 matches (CC is not SGPE):

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>24, 24</td>
<td>12, 48</td>
</tr>
<tr>
<td>D</td>
<td>48, 12</td>
<td>20, 20</td>
</tr>
</tbody>
</table>

Differences in expected length of interaction across treatments are compensated for by differences in payoff discounting
Fréchette and Yuksel — Descriptive Data

Three sessions for each treatment with 18 matches each

Rounds per match:

- RT: 4.5 rounds (1 to 19)
- D + RT: 7.5 rounds (4 to 22)
- BRT: 6.1 rounds (4 to 20)

Average earnings:

- RT: $23.9 ($12.2 to $32.6)
- D + RT: $20.7 ($14.1 to $27.2)
- BRT: $20.9 ($13.6 to $28.7)
Fréchette and Yuksel — Cooperation Rate by Match

Round 1

All Rounds

Cooperation Rate

Match

Cooperation Rate

Match
Fréchette and Yuksel — Effects of Past Observations
Matches 2 to 12, probit marginal effects for probability of cooperation

<table>
<thead>
<tr>
<th></th>
<th>RT</th>
<th>D+RT</th>
<th>BRT</th>
<th>BRT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Partner cooperated in</td>
<td>0.213**</td>
<td>0.172***</td>
<td>0.070</td>
<td>0.068</td>
</tr>
<tr>
<td>Round 1 of previous match</td>
<td>(0.086)</td>
<td>(0.065)</td>
<td>(0.058)</td>
<td>(0.067)</td>
</tr>
<tr>
<td>Number of rounds</td>
<td>0.059*</td>
<td>0.041***</td>
<td>-0.041</td>
<td>-0.037**</td>
</tr>
<tr>
<td>in previous match</td>
<td>(0.032)</td>
<td>(0.013)</td>
<td>(0.028)</td>
<td>(0.018)</td>
</tr>
<tr>
<td>Number of rounds</td>
<td>-0.00334</td>
<td>-0.00333*</td>
<td>0.00288</td>
<td>0.00082</td>
</tr>
<tr>
<td>in previous match sq.</td>
<td>(0.00208)</td>
<td>(0.00179)</td>
<td>(0.00246)</td>
<td>(0.00142)</td>
</tr>
<tr>
<td>Two blocks</td>
<td></td>
<td></td>
<td></td>
<td>0.02277</td>
</tr>
<tr>
<td>in previous match</td>
<td></td>
<td></td>
<td></td>
<td>(0.03918)</td>
</tr>
<tr>
<td>Three blocks</td>
<td></td>
<td></td>
<td>0.1654***</td>
<td></td>
</tr>
<tr>
<td>in previous match</td>
<td></td>
<td></td>
<td>(0.05183)</td>
<td></td>
</tr>
<tr>
<td>Match number</td>
<td>0.000079</td>
<td>0.00279</td>
<td>0.00624</td>
<td>0.00581</td>
</tr>
<tr>
<td></td>
<td>(0.00986)</td>
<td>(0.0153)</td>
<td>(0.0091)</td>
<td>(0.00874)</td>
</tr>
<tr>
<td>Subject cooperated in</td>
<td>0.673***</td>
<td>0.755***</td>
<td>0.350**</td>
<td>0.347**</td>
</tr>
<tr>
<td>Round 1 of match 1</td>
<td>(0.044)</td>
<td>(0.028)</td>
<td>(0.153)</td>
<td>(0.154)</td>
</tr>
<tr>
<td>N</td>
<td>550</td>
<td>528</td>
<td>462</td>
<td>462</td>
</tr>
</tbody>
</table>

Clustered (session level) standard errors in parentheses
Fréchette and Yuksel — Cooperation Evolution

Cooperation rate as a function of the previous choice of the opponent

- Decrease in the probability of cooperation following a defection by the other player and slight increase otherwise
- Difference changes from 46 percent (±2 percent) to nearly twice as much
Fréchette and Yuksel — Cooperation Evolution

Average cooperation for round 1 (solid line), 4 (red dashed), 5 (green dashed):

- **D + RT**: Just before and just after the transition to RT
- **BRT**: End of first block and start of second block

- Stability of cooperation in **D + RT**: more forgiving strategies?
- Slight block restart effect in **BRT**
Fréchette and Yuksel — Strategies

Strategies considered:

- Always Defect: always play $D$
- Grim trigger: play $C$ until either player plays $D$, then play $D$ forever
- Tit-for-Tat: play $C$ unless partner played $D$ last round
- 2-Tits-for-2-Tats: play $C$ unless partner played 2 consecutive $D$s in any of the last 3 rounds
- Suspicious Tit-for-Tat: play $D$ in the first round, then TFT
Fréchette and Yuksel — Strategies

• Strategy $s^k_{imr}(\cdot) \in \{-1, 1\}$ for $D$ and $C$ respectively: Choice that a strategy $k$ indicates to make in round $r$ of match $m$ for subject $i$ given previous history.

• Observed choice $y_{imr} = 1_{\{s^k_{imr}(\cdot) + \gamma \epsilon_{imr} \geq 0\}} \in \{0, 1\}$ for $D$ and $C$ respectively — $\epsilon_{imr}$ independent, logit distribution.

Likelihood that subject $i$ uses strategy $k$:

$$p_i(s^k) = \prod_{M_i} \prod_{R_{im}} \left( \frac{1}{1 + \exp(-s^k_{imr}(\cdot)/\gamma)} \right)^{y_{imr}} \left( \frac{1}{1 + \exp(s^k_{imr}(\cdot)/\gamma)} \right)^{1-y_{imr}}$$

Total log likelihood:

$$\sum_{l} \log \left( \sum_{K} \Phi^k p_i(s^k) \right)$$

where $\Phi$ is the common strategy distribution.
Table 6: Distribution of Estimated Strategies

<table>
<thead>
<tr>
<th>Strategy</th>
<th>RT</th>
<th>D+RT</th>
<th>BRT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Always Defect</td>
<td>0.14</td>
<td>0.26**</td>
<td>0.25***</td>
</tr>
<tr>
<td></td>
<td>(0.098)</td>
<td>(0.107)</td>
<td>(0.072)</td>
</tr>
<tr>
<td>Grim</td>
<td>0.32***</td>
<td>0.10</td>
<td>0.21***</td>
</tr>
<tr>
<td></td>
<td>(0.098)</td>
<td>(0.061)</td>
<td>(0.077)</td>
</tr>
<tr>
<td>Tit-For-Tat</td>
<td>0.39***</td>
<td>0.22**</td>
<td>0.33***</td>
</tr>
<tr>
<td></td>
<td>(0.118)</td>
<td>(0.095)</td>
<td>(0.089)</td>
</tr>
<tr>
<td>2 Tits-For-2 Tats</td>
<td>0.06</td>
<td>0.06***</td>
<td>0.07*</td>
</tr>
<tr>
<td></td>
<td>(0.044)</td>
<td>(0.021)</td>
<td>(0.043)</td>
</tr>
<tr>
<td>Suspicious Tit-For-Tat</td>
<td>0.02</td>
<td>0.18***</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>(0.061)</td>
<td>(0.057)</td>
<td>(0.036)</td>
</tr>
</tbody>
</table>
Fréchette and Yuksel — Summary

• All 3 methods can be used to induce infinitely repeated games.
• Cooperation rates are highest with RT despite fewer interactions.
• RT and BRT: Somewhat similar.
• RT to D + RT: More Always Defect, Suspicious Tit-for-Tat, less Grim and less Tit-for-Tat.
• Situations in which agents are very patient, but relationships are likely to terminate for exogenous reasons, may lead to different strategic choices, and consequently different dynamics than situations in which agents are less patient, but interactions are less likely to end.