Lecture Outline:

I. New Notation

II. Types of Systems
   a. Hierarchical
   b. Recursive

III. Two Stage Least Squares and Indirect Least Squares

IV. Identification

V. Simultaneous Equations
   a. Gender Attitudes
   b. Supply and Demand
   c. Peer Effects

Notation. In order to understand systems of equations we need to use some matrix notation. Let:

Y = (n by s) matrix of a dependent or endogenous variables.
X = (n by k) matrix of independent or exogenous variables.
B = (k by s) matrix of slope coefficients
A = (s by s) matrix of slope coefficients
E = (n by s) matrix of errors.

Now consider the simplest example of a system of s equations which generalizes the single equation regression model: $Y = XB + E$. This is similar to our usual model
except that we have $s$ equations instead of just 1. Writing it out in the transposed form we have:

$$
Y_{i1} = (X_{i1}b_{11} + \ldots X_{ik}b_{1k}) + e_{i1} \\
Y_{i2} = (X_{i1}b_{12} + \ldots X_{ik}b_{2k}) + e_{i2} \\
\vdots \\
Y_{is} = (X_{i1}b_{1s} + \ldots X_{ik}b_{ks}) + e_{is}
$$

What we have here is just $s$ equations for $s$ different $Y$'s where in each case the $Y$ is some linear function of the $X$'s. In this example the $Y$'s are only functions of the $X$'s and not of each other. The typical case where this type of system arises is when we write a system of equations in reduced form. In general we want to consider models in which there is interdependency between the $Y$'s also. The simplest case is where we have a hierarchical structure. Then we have: $Y = YA + XB + E$ where $A$ is of the form:

$$
A = \begin{bmatrix}
0 & a_{12} & a_{13} & \ldots & a_{1s} \\
0 & 0 & a_{23} & \ldots & a_{2s} \\
\vdots & & & & \ddots \\
0 & 0 & 0 & \ldots & a_{s-1s} \\
0 & 0 & 0 & \ldots & 0
\end{bmatrix}
$$

Technically, we describe this as $A$ being upper triangular. We can write this out in transposed form in terms of the following equations:
\[ Y_{i1} = X_i B_1 + E_{i1} \]
\[ Y_{i2} = Y_{i1} A_{i2} + X B_2 + E_{i2} \]
\[ \ldots \]
\[ Y_{iS} = Y_{i1} A_{1S} + Y_{i2} A_{2S} \ldots Y_{S-1} A_{S-1S} + X_i B_S + E_{iS} \]

Notice that \( Y_{i1} \) is only a function of the \( X \)'s. \( Y_{i2} \) is a function of \( Y_{i1} \) and the \( X \)'s and so on until we get to \( Y_{iS} \) which is function of all the other \( Y_{iS}' \). (Some of the \( a \)'s could be zero, but that is not important here.) There is in a sense a hierarchy with \( Y_1 \) on top and \( Y_s \) at the bottom and with each \( Y \) potentially effecting those \( Y \)'s that are below it.

The importance of hierarchical structures is that they assume that there is an explicit causal ordering to the variables. \( Y_1 \) is causally prior to all other \( Y \)'s, \( Y_2 \)'s is causally prior to all the other \( Y \)'s except \( Y_1 \) and so on. No \( Y \) that is lower down in the hierarchy either directly or indirectly affects any \( Y \) higher up. In this sense, the causal ordering is one way. **The system is nonsimultaneous.** No two \( Y \)'s mutually affect each other, either directly or indirectly.

We can put the expression \( Y = YA + XB + E \) in reduced form, endogenous variables on the left, only exogenous variables on the right side of the equation, by using standard algebraic manipulations:

\[ Y = YA + XB + E \text{ rearranging terms} \]
\[ Y(I-A) = XB + E \text{ multiply by } (I-A)^{-1} \]
\[ Y = XB(I-A)^{-1} + E(I-A)^{-1} = XT + EV \text{ reduced form} \]
The conventional theory of path analysis only concerns itself with hierarchical systems. The rules we discussed earlier in the course for calculating indirect and total effects from paths also only apply to hierarchical systems. Traditional path models in fact have one additional property -- the errors across equations are assumed to be independent. Hierarchical systems of equations with this property are called recursive. The importance of recursive systems is that they can be estimated using ordinary least squares. Consider one particular equation:

\[ Y_{im} = Y_{i1}A_{im} + Y_{i2}A_{2m} \ldots Y_{im-1}A_{m-1m} + X_iB_m + E_{im} \]

In order to get unbiased and consistent estimates of the A's and B's in this equation, the major condition that has to be met is that the Y's and X's in the equation have to be uncorrelated with the error term. Now assume that the error term here is correlated with the error in the equation predicting \( Y_j \) with \( j < m \). Since \( E_j \) and \( E_m \) are assumed to be correlated and \( Y_j \) is a function of \( E_j \), then \( E_m \) and \( Y_j \) must be correlated, but then we could not estimate the equation above using OLS.

Thus, we see that the assumption of having uncorrelated errors is quite important if we want to use OLS to estimate our slope coefficients. It is also a very strong assumption. It is saying that we believe that there are no common omitted variables across the equations in our path model. Thus in our earlier example (Lecture 6) where we had one equation in which we predicted son's education from father's education and another equation in which we predicted son's occupation from son's education and
father's education, by using OLS we were implicitly assuming that there were no omitted variables common to the two equations.

Now consider systems that are hierarchical, but in which we drop the assumption that the error terms are uncorrelated. As such OLS is no longer feasible and other methods of estimation are needed. There are three different approaches to dealing with this situation: instrumental variables, indirect least squares, and two-stage least squares.

Indirect Least Squares. Indirect Least Squares simply involves estimating the reduced form equations and then solving out for the structural parameters from the reduced form parameters.

From above we have:

\[
Y = Ya + Xd + E \quad \text{rearranging terms}
\]
\[
Y(I-A) = Xd + E \quad \text{multiply by } (1-A)^{-1}
\]
\[
Y = Xd(I-A)^{-1} + E(I-A)^{-1} = XT + EV \quad \text{reduced form}
\]

Thus, we have that \(B(I-A)^{-1} = T\). As I discuss below, a key issue is whether it will be possible to solve for \(A\) and \(B\) from the estimates of \(T\).

Two Stage Least Squares. In lecture 11, I briefly discussed 2SLS. Let me review it here using the notation from that lecture. Assume that we have two instruments for \(T\). Consider the following path model:
\( \hat{T} \) is an instrument for \( T \) since it is unassociated with either of the errors \( U \) and does not directly affect \( Y \). This is the idea behind two stage least squares.

Stage I: Estimate using OLS:

\[
T = Z_1c_1 + Z_2c_2 + U_t
\]

Calculate:

\[
\hat{T} = Z_1\hat{c}_1 + Z_2\hat{c}_2
\]

Stage II: Estimate using OLS:

\[
Y = \hat{T}b + U_t b + U_y = \hat{T}b + w
\]

Notice that in Stage II it is legitimate to use OLS because \( \hat{T} \) is uncorrelated with neither \( U \) by construction and thus uncorrelated with \( w \). Often Two Stage Least Squares (TSLS) is described as purging \( T \) of the component of it that is associated with the \( U \)'s (Stage I) and then estimating the effect of \( T \) on \( Y \) by using the purged \( T \), \( \hat{T} \) (Stage II). A word of caution. Since 2SLS is a form of IV, we must use
the IV formula for the standard errors that we derived above: \( \text{Var}(\hat{b}_n) = \sigma^2 \left( Z'X \right)^{-1} Z'Z(Z'Z)^{-1} \).

In terms of the notation being used in this lecture, \( T \) would be a \( Y \) since in the above diagram it is endogenous. That is, in the above diagram there would be two \( Y \)'s, the \( Y \) that is the final outcome, call it \( Y_2 \), and \( T \), which we might label, \( Y_1 \).

Under specific conditions, IV, 2SLS, and Indirect Least Squares will be equivalent.

Theorem 1. Two stage least squares is equivalent to Instrumental Variables with \( \hat{Y} \) as the instrument.

Theorem 2. Instrument variables is equivalent to two stage least squares if there is one instrument per variable and \( \hat{Y} \) has been predicted from its instrument.

Theorem 3. Indirect least squares is equivalent to 2SLS and IV when there is only one instrument per variable.

To clarify things consider the following example.

\[
\begin{align*}
\text{Ed} & = \text{Father's Ed } a_1 + u_1 \\
\text{Ability} & = \text{F's Ability } a_2 + u_2 \\
\text{ln Wage} & = \text{Ed } b_1 + \text{Ability } b_2 + e
\end{align*}
\]

We could represent these equations in terms of the following path model:
where the dotted lines represent possible additional effects. Assume that we believe that the error terms are correlated across equations or equivalently that in the last equation there are X's that have been omitted that are correlated with Education and Ability so that it is not possible to use OLS to estimate the last equation. Assume, however, that the first two equations are well specified in that Father's Ed and Father's Ability are not correlated with any variable that has been omitted from either of the three equations. Now consider the following three approaches to estimating the ln Wage equation.

Instrumental Variables:

Estimate using Father's Ed and Father's Ability as instruments for Ed and Ability.

Indirect least squares: Estimate using OLS

\[
\begin{align*}
\text{Ed} &= \text{Father's Ed } a_1 + u_1 \\
\text{Ability} &= \text{F's Ability } a_2 + u_2 \\
\ln \text{Wage} &= \text{Father's Ed}(a_1 b_1) + \text{F's Ability}(a_2 b_2) + u_1 b_1 + u_2 b_2 + e
\end{align*}
\]

and solve for

\[
\hat{b}_1 = (a_1 b_1)/\hat{a}_1 \quad \hat{b}_2 = (a_2 b_2)/\hat{a}_2
\]
Two Stage Least Squares:

Stage 1: Estimate Using OLS:

\[
\text{Ed} = \text{Father's Ed} a_1 + u_1 \\
\text{Ability} = \text{Father's Ability} a_2 + u_2
\]

Stage 2: Estimate Using OLS

\[
\hat{\ln \text{ Wage}} = \hat{\text{Ed}} b_1 + \hat{\text{Ability}} b_2 + u_1 b_1 + u_2 b_2 + e
\]

In the present case because we have the same number of instruments as variables all three procedures will give us the same estimates.

Identification. I now want to talk about the problem of identification in terms of indirect least squares and two stage least squares. Consider estimating a system in terms of indirect least squares. We start by estimating the reduced form equation \( Y = X^T + U \) using OLS. We then want to solve for the coefficients of A and B. In this case we have the problem as to whether we have as many equations as unknowns. Put another way, we need to know whether we have enough information to identify all the coefficients in A and B. Three situations are likely to occur. First, we will have fewer equations than unknowns in which case we will not be able to solve for all the structural coefficients. Technically this is what is known as having an underidentified model. Consider the model:

<table>
<thead>
<tr>
<th>Structural Model</th>
<th>Reduced Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X = Za + e_1 )</td>
<td>( X = Za + e_1 )</td>
</tr>
<tr>
<td>( Y = Xb_1 + Zb_2 + e_2 )</td>
<td>( Y = Z(b_2 + ab_1) + ab_2 e_1 + e_2 )</td>
</tr>
</tbody>
</table>
In the reduced form we only estimate 2 slope coefficients \( a \) and \( (b_2+b_1a) \), but we have three unknown coefficients - \( a \), \( b_1 \) and \( b_2 \). Clearly, the model is underidentified - we have two equations in three unknowns.

The second possibility is that we have as many equations as unknowns and are able to solve for the unknowns from the equations. In this case the model is called just identified. We have just enough information to identify all the coefficients. In the example above if we set \( b_2 = 0 \) (by assumption) we would have two equations and two unknowns and the model would be just identified.

Finally, we can have situations in which we have more equations than unknowns and have more information than is needed to identify the models. This will often occur when there are more instruments than there are variables needing them. In this situation we say the model, or more precisely, a parameter is overidentified.

A necessary condition for identification is that there be as many reduced form parameters as there are structural coefficients to estimate. Unfortunately, this is not a sufficient condition for identification. The sufficient condition for identification is quite complicated and generally not of much practical use. It is discussed at length in Hanushek and Jackson. You are welcome to read their treatment of the problem. You are not, however, responsible for it.

A preferable approach that works in almost all situations is to consider the problem in terms of two stage least squares. Assume that we estimate our equations as follows:
\[ \hat{Y} = XT + U \quad \text{Using OLS \quad Stage 1} \]
\[ Y = \hat{YA} + XB + E \quad \text{Using OLS \quad Stage 2} \]

Note that the second set of equations can be estimated using OLS since \( X \) is uncorrelated with \( E \) by assumption and thus \( \hat{Y} \) is uncorrelated with \( E \) by construction. Notice also that if the model is overidentified two stage least squares takes care of this by pooling all the information in \( T \) into \( \hat{Y} \). It can be shown that two stage least squares actually does this in an optimal way.

Now consider the question as to whether the second equation in this two stage process is estimable in the sense that the \( X \) and \( \hat{Y} \) are linearly dependent. This can be determined on an equation basis. Note that given that the \( \hat{Y} \)'s are linear functions of the \( X \)'s, there are real potentials for problems here. Consider one particular equation:

\[ Y_j = \hat{Y}_1A_{1j} + \hat{Y}_2A_{2j} + \ldots + \hat{Y}_{j-1}A_{j-1} + XB_j + E_j \]

Without making further restrictions, the \( \hat{Y} \)'s in this equation are going to be linear functions of the \( X \)'s and thus each \( \hat{Y} \) will be linearly dependent on the \( X \)'s and the equation will not be estimable. Since the \( \hat{Y} \)'s are a function of the \( X \)'s a necessary condition for identifiability is that there be one \( X \) for each \( \hat{Y} \) such that \( \hat{Y} \) is a function of that \( X \) in the reduced form equation, but that \( X \) does not enter into the structural equation under consideration. That is for each \( \hat{Y} \) there must be an \( X_i \) such
that $B_{1j} = 0$. Each $\hat{Y}$ must depend on some X which is not in the structural (2nd Stage) equation in order for the equation to be estimable. Such X's are by definition instruments for the Y's. They have a direct effect on the "independent variable" of concern, one of the Y's, yet they do not directly affect the dependent variable in the equation. A necessary condition is thus that each Y on the right hand side of the equation have an instrument. Note here that an instrument is very easy to define since all it is is an X that affects Y in the reduced form equation $Y = XT + U$ but does not enter into the structural equation where that Y is an independent variable ($B_{1j} = 0$).

This condition, however, is not quite strong enough. Assume that we had two Y's in an equation and they both had the same instrument $X_1$. In this case the two Y's would not linearly depend on the X's in the equation since they were a function of an X that had been excluded from the equation, but they would be linearly dependent on each other since the part of each which was independent of X in the equation depended on the same excluded X. A condition that is sufficient for ruling out the possibility that the Y's are linearly dependent on each other is that there be at least as many instruments as Y's for any subset of Y's on the right hand side of the equation.

At first sight this condition may seem odd. However, consider the possibility that we have two Y's one of which has two instruments and one of which has none. Clearly, we are not going to be able to estimate a coefficient for the second Y since it is a linear function of the X's. Notice, however, that we do have two equations from the two instruments and two unknowns. What has happened is that
some of the coefficients in the model are overidentified
(the coefficient for the variable with two instruments) and
some are underidentified (the variable with no instrument).

The rule that each subset of Y's on the right hand
side of an equation must have as many instruments as
variables rules out this last possibility. From our
discussion we then have the following set of rules for
identification.

1. Check to see if there are as many reduced form
parameters to be estimated as there are structural
parameters to be estimated.

2. Check to see for each equation if each endogenous
variable on the right side of the equation has an
instrument.

3. Check to see that each subset of endogenous variables on
the right hand side of the equation has as many instruments
as variables.

In most situations these rules should suffice. There are
somewhat weaker conditions under which a model can be
identified. This almost never occurs. See Hanushek and
Jackson for a discussion of necessary conditions for
identification.

In the above discussion we have ignored error terms.
In general it is undesirable to make assumptions about the
structure of the error terms. If we do, it is sometimes
possible to identify a model which is not identified with
respect to just the coefficients on X in the reduced form.

Simultaneous Equations. Our three approaches
(instrumental variables, indirect least squares, and two
stage least squares) to estimating equations where our
independent variables are correlated with our error term,
can also be used to estimate systems of equations that are nonhierarchichal, that is, some of the Y’s simultaneously affect each other. There is little new to be learned here, but looking at some examples is of interest. All the examples are of nonrecursive, nonhierarchichal models.

Consider the following example taken from the Duncans' book Sex Typing and Social Roles:

The Duncans were interested in the effect of spouse's attitudes regarding sex roles on each other. The figure above shows one possible model. We have the education of each spouse effecting how liberal their attitudes are towards the appropriateness of children of each sex carrying out a particular job -- in this case washing a car. For the moment let us ignore the results of their analysis and focus on the problem of estimating the above system of equations. The structural equations represented by the above diagram are:

\[
\begin{align*}
HA &= a_1WA + a_2HE + e_1 \\
WA &= b_1HA + b_2WE + e_2
\end{align*}
\]
We cannot estimate these two equations using OLS. First, note that part of the relation between spouses' attitudes is due to the effect of HA on WA and part is due to the effect of WA on HA. If we just estimate the equations separately then in this case we would end up overstating the effect of each attitude on the other since we wouldn't be taking account of the fact that the causality was working in both directions. Second, note that spouse's attitude is going to be correlated with the error term in each equation. The easiest way to see this is to substitute the two equations into each other getting:

\[
\begin{align*}
HA &= (a_1b_1HA + a_1b_2WE + a_1e_2) + a_2HE + e_1 \\
WA &= (b_1a_1WA + b_1a_2HE + b_1e_1) + b_2WE + e_2
\end{align*}
\]

Both spouses' attitudes are a function of both error terms. Thus in the structural equations above in the first equation WA will be correlated with \(e_1\) and in the second equation HA will be correlated with \(e_2\). If we estimated the two structural equations using OLS we would get biased results since one of our independent variables is correlated with the error term.

Note that there is a potential instrument for HA and WA, the education of each. The Duncans have assumed that the education of a person has no direct effect on his/her spouse's attitude. Presumably, though, we observe a correlation between a person's education and his spouses attitude. We can use this information to identify the model. In practice, we could either use instrumental variables, indirect least squares or two stage least squares. Consider the reduced form equations that we would
use in indirect least squares (note that these can be gotten from the equations above):

\[
HA = \frac{a_1b_1}{1-a_1b_1} WE + \frac{a_2}{1-a_1b_1} HE + u_1
\]

\[
WA = \frac{b_1a_2}{1-a_1b_1} HE + \frac{b_2}{1-a_1b_1} WE + u_2
\]

If we estimate the reduced form equations, we will have four coefficients and thus four equations. We also have four unknowns \((a_1, a_2, b_1, b_2)\) so we should be able to identify the model. Looking at the structural equations (or the reduced form equations for that matter), we see that there are two endogenous variables \((HA \text{ and } WA)\). Each of these two variables has a distinct instrument \((HE \text{ and } WE \text{ respectively})\) for the equation in which they enter as an independent variable, so using our stricter rule for identification, the model is also identified. We can show that the model is actually identified by expressing the structural coefficients as functions of the reduced form coefficients.

Consider another example - the classic supply and demand equations of economics. It is here that the two equation simultaneous equation system got its start in the social sciences. We have two graphs which we can draw as one.
In the first graph we have the relationship between supply and price. The higher the price of something, the more manufacturers are willing to make of it. In the second graph we have the relationship between demand and price. The cheaper a product the more of it that consumers will buy. In the final graph we have the two graphs drawn together. If we accept the ideas of classical economics then the amount actually produced should be equal to the amount bought which is represented by the intersection of the demand and supply curves. This is just a single point. If we only have information on the quantity sold at a specific price then we will not be able to estimate either the supply or demand equations since all we observe is the single quantity price point representing the intersection of the demand and supply curves. To see the problem more deeply write out the equations:

Demand: \[ Q = a_1 + a_2P + e_1 \]

Supply: \[ Q = b_1 + b_2P + e_2 \]

\[ P = c_1 + c_2Q + e_2 \]
In the case of the supply equation, I have rewritten the equation so that price is the dependent variable so that the formulation parallels the sex role attitude example. In this case, in the demand equation we have quantity being determined by price and in the supply equation we have price being determined by quantity. Thus, there are two endogenous variables. Neither variable, however, has an instrument, so the system is underidentified. Assume now that manufacturers in different markets face different costs with respect to materials and because of this the supply schedule differs in these markets. Let us represent this as follows:

\[
\text{Demand: } Q = a_1 + a_2P + e_1
\]

\[
\text{Supply: } P = c_1 + c_2Q + c_3C + e_2
\]

Notice that the variable C representing different costs of material will be an instrument for the variable P in the demand equation. Thus the demand equation is now identified.

To see why this is graphically note that what C does is shift the supply curve up and down. This is represented by the dotted supply curves in the graph of the supply curve on the last page. Notice what the effect of having this set of supply curves is when we combine this information with the demand curve in the last curve. By observing markets with different supply curves, we will observe different points along the demand curve as the intersection between the demand and supply curves for each market differs. In essence, the different supply curves across markets allows us to trace out the demand curve.
This shows graphically why having an instrument for Price in the demand curve allows us to identify the model.

As a final example consider the more complicated model presented in H and J for occupational and educational aspirations which they take from the work of Duncan, Haller, and Portes. This is shown on the last page.

We need to first ask whether the model is identified. There are three pairs of equations in the model: two parental aspiration equations (P1 and P2), two occupational aspiration equations (O1 and O2), and two educational aspiration equations (E1 and E2). Certainly if one equation in each pair is identified the other will be since the model is perfectly symmetrical.

Consider the parental aspiration equation for the respondent (P1). P1 is just a function of IQ (I1) and socioeconomic background (S1) which are both exogenous variables so there is no problem with the equation being identified.

Now consider the occupational aspirations equation (O1). O1 is a function of IQ (I1), socioeconomic background (S1), parental aspirations (P1) and friends occupational aspirations (O2). I1 and S1 are exogenous so there is no problem there. P1 is endogenous so it needs an instrument. Looking at the diagram we see that P1 doesn't have an instrument since there is no variable that effects it that doesn't effect O1. Thus, we will not be able to identify the effect of P1 on O1.

Next, O1 is a function of O2, friend's occupational aspirations. There are two possible instruments for O2, I2, friend's I.Q. and S2, friend's socioeconomic status. Thus, we should be able to identify the effect of O2 on O1.
Finally, consider the equation for $E_1$, respondent's educational aspirations. $E_1$ is a function of $I_1$ and $S_1$ which are exogenous. It is also a function of $E_2$ and $O_1$ which both have as instruments $I_2$ and $S_2$, so their coefficients should be identified. Finally, it is a function of $P_1$ which as before has no instrument and whose effect is thus not identified.

The table below the path diagram shows the OLS and 2SLS estimates of our equations. Parental aspirations has been left out of the occupational and educational aspirations equations. Implicitly, we are controlling for parental aspirations in these two equations by controlling for the effects of IQ and socioeconomic status. Thus the coefficients on these variables represent the direct effect of these variables plus their indirect effect through parental aspirations, i.e. their total effects.

In the parental aspiration equation the OLS and 2SLS estimates are identical. For the other two equations we get substantially different results. Of particular note is the size of the peer aspirations effects. These are much larger in the 2SLS estimates than for the OLS estimates. This is somewhat counterintuitive. It probably can be explained by the fact that instrumental variables have taken care of not only the simultaneity problem, but also measurement error in the peer occupation and educational aspiration variables which would have downwardly biased their coefficients.
I - INTELLIGENCE
S - SOCIOECONOMIC BACKGROUND
P - PARENTAL ASPIRATIONS
O - OCCUPATIONAL ASPIRATIONS
E - EDUCATIONAL ASPIRATIONS
U's ARE ERROR TERMS WHICH ARE POTENTIALLY CORRELATED.

WE HAVE ESSENTIALLY TWO SAMPLES: A SAMPLE OF RESPONDENTS (TOP PART OF MODEL) AND THEIR BEST FRIENDS (BOTTOM PART OF MODEL).

![Simultaneous peer influence model](image)

TABLE 9.1
Estimates of Limited Occupational and Educational Aspiration Equations

<table>
<thead>
<tr>
<th></th>
<th>Respondent</th>
<th>Friend</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TSLS</td>
<td>OLS</td>
</tr>
<tr>
<td>Parental aspirations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intelligence</td>
<td>0.11</td>
<td>(0.04)</td>
</tr>
<tr>
<td>Status</td>
<td>-0.01</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Occupational aspirations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intelligence</td>
<td>0.643</td>
<td>(0.124)</td>
</tr>
<tr>
<td>Status</td>
<td>0.348</td>
<td>(0.113)</td>
</tr>
<tr>
<td>Occupational Aspirations—Peer</td>
<td>0.409</td>
<td>(0.104)</td>
</tr>
<tr>
<td>Educational aspirations—Peer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intelligence</td>
<td>0.050</td>
<td>(0.039)</td>
</tr>
<tr>
<td>Status</td>
<td>0.057</td>
<td>(0.033)</td>
</tr>
<tr>
<td>Occupational Aspirations</td>
<td>0.027</td>
<td>(0.060)</td>
</tr>
<tr>
<td>Educational Aspirations—Peer</td>
<td>0.197</td>
<td>(0.264)</td>
</tr>
</tbody>
</table>

*Estimated standard errors appear in parentheses.*

21