Introduction

Last few lectures:

- Consumption with precautionary savings, borrowing constraints (and behavioral frictions).
- Important insights but in partial equilibrium.
- **Today:** Consumption in general equilibrium with BC.
  - Are partial equilibrium insights robust?
  - Are there additional insights from a GE analysis?

- **Methodological:** Introduction to neoclassical macro with incomplete markets (and heterogeneous agents).
Why should we study general equilibrium?

Consider the income fluctuations problem with i.i.d. income shocks.

- Need $r < \delta$ to have buffer-stock behavior.
- What happens if $r \approx \delta$? Does this sound reasonable?
- **Equilibrium in the asset market:** Endogenous determination of $r$.
- The level of $r$ also important input into the liquidity trap.

Consider a credit crunch and a drop in consumption:

- Productive capacity of the economy is unchanged.
- Why/how does this lower output in GE?
- **Equilibrium in the goods market:** Endogenous output.

For a better understanding, need to study general equilibrium
Today’s plan


1. Endogenous determination of $r$.
2. Effect of a credit crunch on $r$.


1. Transition to new steady state.
2. Quantitative significance for the recent crisis.
3. Endogenous determination of output (as well as $r$).
Aiyagari (1994): Environment is neoclassical

- Economy with production function, $f(k, l)$.
- Continuum of measure one of consumers, denoted by $i$.
- Labor endowment shocks (i.i.d.). Distribution, $dF$, with bounded support $[l_{\text{min}}, l_{\text{max}}]$. Mean normalized to 1.
- Preferences:
  $$E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_{it}) \right].$$
- Initially all consumers have the same level of assets, $a_0$.

Suppose asset markets are complete. How do we characterize the equilibrium?
Equilibrium with complete markets

- All idiosyncratic risks are insured. Consumption \( c_{it} \equiv c_t \) for each \( i \).
- Using FWT, equilibrium characterized as the solution to:

\[
\max_{\{c_t, k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t u(c_t),
\]

s.t. \( c_t + k_{t+1} = k_t (1 - \delta) + f(k_t, 1) \).

- Steady state, \((c^*, k^*)\), is the solution to:

\[
u'(c^*) = \beta (1 + r) u'(c^*) \text{ where } r = f_1(k^*, 1) - \delta,
\]

and \( \delta k^* + c^* = f(k^*, 1) \).

- In particular \( \beta (1 + r) = 1 \), which implies:

\[
\lambda \equiv r^* = 1/\beta - 1.
\]
Key assumption: Uninsured idiosyncratic risks

What if markets are incomplete?

- **Extreme case:** Only riskless asset is available.
- Then, consumers face the income fluctuations problem.
  - Labor income, $w_{lt}$, is uncertain.
  - Can hold assets (or borrow up to a limit) at rate $r$.
- Key difference from Deaton: $w$ and $r$ are endogenous.
Consider the problem of one consumer with $l_{it}$.

- **Budget constraint:**
  \[ c_{it} + a_{it+1} = w_{it} + (1 + r) a_{it}. \]

- **Borrowing constraint:**
  \[ a_{it+1} \geq -\phi, \text{ where } \phi = \min \begin{cases} \frac{b}{\text{exogenous}}, & \frac{w_{it\min}}{r} \end{cases} \text{ endogenous}. \]

- **Note:** Convention for $a_{it+1}$ (timing) slightly different than in Deaton.
Consumer’s problem

- Define **cash-in-hand** as:
  \[ z_{it} = w l_{it} + a_{it} (1 + r) + \phi. \]

- Define **slack** from debt limit as:
  \[ \hat{a}_{it+1} = a_{it+1} + \phi. \]

- Consumers’ constraints become:
  - Budget constraint: \( c_{it} + \hat{a}_{it+1} = z_{it} \),
  - Borrowing constraint : \( \hat{a}_{it+1} \geq 0. \)
  - Evolution of cash-in-hand: 
    \[ z_{it+1} = w l_{it+1} + \hat{a}_{it+1} (1 + r) - r\phi. \]
Consumer’s problem

- Bellman equation:

\[
V(z_{it}, \phi, w, r) = \max_{\hat{a}_{it+1} \in [0, z_{it}]} u(z_{it} - \hat{a}_{it+1}) + \beta \int V(z_{it+1}, \phi, w, r) dF(l_{it+1}),
\]

s.t. \( z_{it+1} = wl_{it+1} + (1 + r) \hat{a}_{it+1} - r\phi. \)

- Denote the solution with:

\[
\hat{a}_{it+1} = A(z_{it}, \phi, w, r).
\]

Characterization similar to Deaton.
Figure 1a
Consumption and Assets as Functions of Total Resources
This picture implies a stable invariant distribution for $z_{it}$. 
Cross-sectional cash distribution converges

- The policy $A(\cdot)$ generates an endogenous process for $z$:

$$z_{t+1} = w^* l_{t+1} + (1 + r) A(z_t, \phi, w, r) - r \phi.$$  

Starting with some distribution $dG_0(z)$, we have:

$$dG_0(z) \rightarrow^{A(\cdot)} dG_1(z) \rightarrow^{A(\cdot)} ...$$

Under regularity assumptions, the distribution converges to a steady-state, $dG(\cdot)$. 
Define **net asset demand** in steady-state as:

\[ E_{a_{w}}(r) \equiv \int (A(z, \phi, w, r) - \phi) \, dG(z). \]

- This is increasing in \( r \). Why?
Definition of equilibrium with incomplete markets

A steady-state equilibrium consists of a policy function, $A(z, \phi, w^*, r^*)$, a steady-state distribution, $dG(z)$, capital stock $k^*$, and prices, $\{r^*, w^*\}$, such that:

1. Policy function is optimal given $w^*$ and $r^*$.
2. The steady-state distribution is consistent with the policy function.
3. Capital, labor, and asset markets clear:

$$r^* = f_1(k^*, 1) - \delta \quad \text{and} \quad w^* = f_2(k^*, 1) \quad (1)$$

$$k^* = E_{a_w^*}(r^*).$$

**Important property:** Aggregates are deterministic but individual allocations are not.
Characterizing the equilibrium

- Three unknowns, \((k^*, r^*, w^*)\), and three equations in (1).
- Substitute \(K\left(\begin{array}{c} - \\ r \end{array}\right)\) and \(w\left(\begin{array}{c} - \\ r \end{array}\right)\) into asset market clearing to get:

\[
\begin{aligned}
K(r) &= E_a(r) = E_{aw(r)}(r) \\
&= \text{firms' asset supply} = \text{consumers' net asset demand}
\end{aligned}
\]

- Asset demand equation typically increasing in \(r\), but not always. Why?
- Equilibrium is the (typically unique) intersection.
- Equilibrium \( r \) is always below the full insurance benchmark (\( e^f \)).
- **Deaton’s condition** \( r < \lambda \) is endogenously satisfied. Why?
Credit crunch (e.g., $b > 0$ to $\hat{b} = 0$) typically raises asset demand. Why?
Credit crunch further lowers the interest rate. Why?
Bewley/Aiyagari/Huggett style models important branch of macro:

- More realistic(?) models with heterogeneous agents.
- Particularly appropriate to study long run and distributional issues.
- Can introduce aggregate shocks, but lose tractability. Why?
- Nonetheless, computationally tractable (Krussel and Smith, 1998).
- Methodology: Calibration and numerical simulations.
- For a survey, see Heathcote, Storesletten, Violante (2009).
What the Aiyagari model can and cannot do

Aiyagari model useful to understand $r$, but cannot explain output fluctuations.

- **Full factor utilization** $\implies$ Output is supply determined.
- At impact, credit crunch does not affect output, $y = f(k^{old}, 1)$
- At the new ss, output (and even consumption) is actually **higher**, $y = f(k^{new}, 1)$ and $k^{new} > k^{old}$.

Model intended for the long run. Not very helpful with the business cycle.

**Next:** Guerrieri-Lorenzoni (2011) to focus on short run effects.

- Transitional dynamics to the new steady state.
- Quantitative significance for the recent crisis.
- Output/consumption response (not clear for the right reasons).
Guerrieri-Lorenzoni (2011): Key features

- Uninsurable income shocks.
- No capital (Bewley variety). Firms’ asset supply exogenous.
- **Endogenous labor supply**: Endogenous factor utilization.
- Exogenous borrowing limit.

Focus: **Transitional dynamics** following a **tightening of the limit**.
Continuum of measure 1 of households, denoted by $i$.

Preferences:

$$\max E_0 \left[ \sum_{t=0}^{\infty} \beta^t u \left( c_{it}, n_{it} \right) \right].$$

Linear technology:

$$y_{it} = \theta_{it} n_{it},$$

where $\theta_{it}$ is the idiosyncratic productivity shock.

Productivity, $\theta_{it}$, follows a Markov chain on $\{\theta^1, ..., \theta^S\}$, with $\theta^1 = 0$ (unemployment).
Budget constraint:

$$\frac{a_{i,t+1}}{1 + r_t} + c_{it} + \tilde{\tau}_{it} \leq a_{i,t} + y_{it}.$$  

**lump=sum taxes**  

**asset (bond) holdings**

Exogenous borrowing constraint:

$$a_{i,t+1} \geq -\phi_{t+1}.$$  

Note: Timing convention for $a_{t+1}$ back to Deaton.

Tax policy:

$$\tilde{\tau}_{it} = \begin{cases} 
\tau_t - v_t, & \text{if } \theta_{it} > 0, \\
\text{unemp. benefits}, & \text{if } \theta_{it} = 0.
\end{cases}$$
Government’s budget constraint:

$$B_t + v_t u = \frac{B_{t+1}}{1 + r_t} + \tau_t,$$

where $u = \Pr(\theta_{it} = 0)$ is the fraction unemployed and $v_t$ unemployment benefits.

Baseline: Government keeps $B_{t+1} = \bar{B}$ and $v_t = \bar{v}$ constant. Taxes balance the budget.

Strictly speaking, $\bar{B}$, is government bonds. But it is more broadly interpreted as asset supply from outside consumers (firms plus gov).
Defining the equilibrium

- To consider transitional dynamics, need to go beyond the steady-state.
- Given \( \{b_t\}_t, \{r_t\}_t, \) and \( \{\tau_t\}_t \), there is a solution to consumer’s problem. Denote by \( \{C_t(a, \theta), N_t(a, \theta)\}_t \).
- The solution also pins down a policy function for assets: \( A_t(a, \theta) \).
- Let \( dG_t(a, \theta) \) denote the joint distribution of assets and productivity at date \( t \).
- Then, the policy function (and the \( \theta \) process) generate an endogenous process for \( (a, \theta) \), which implies a mapping:

\[
dG_t(a, \theta) \rightarrow A_t(\cdot) \ dG_{t+1}(a, \theta).
\]

We are now ready to define the equilibrium (not necessarily steady-state).
Defining the equilibrium

Given an initial distribution, $dG_0(a, \theta)$, and a path of borrowing limits, \{\phi_t\}_t, an equilibrium is a collection of interest rates, \{r_t\}_t, policy functions \{C_t(a, \theta), N_t(a, \theta), A_t(a, \theta)\}_t, taxes \{\tau_t\}_t, and distributions \{dG_t(a, \theta)\}_t such that:

1. Policy functions are optimal given \{b_t\}_t, \{r_t\}_t, and \{\tau_t\}_t.
2. The distributions, \{dG_t(a, \theta)\}_t, are consistent with policy functions.
3. The tax rate balances the gov budget: $\tau_t = \bar{\nu}u + \bar{B} \frac{r_t}{1+r_t}$.
4. The asset market clears,

$$\int a \ dG_t(a, \theta) = \bar{B} \text{ for each } t.$$
Insights from optimality conditions

- **Optimality conditions:**
  - Euler equation:
    \[ u_c(c_{it}, n_{it}) \geq \beta (1 + r_t) E_t [u_c(c_{it+1}, n_{it+1})], \text{ with eq. if } a_{it+1} > -\phi_{t+1}. \]
  - Labor choice:
    \[ \theta_{it} u_c(c_{it}, n_{it}) \leq -u_n(c_{it}, n_{it}), \text{ with equality if } n_{it} > 0. \]

- **Insights:**
  1. Keeping \( \{r_t\}_t \) constant, tightening of constraint reduces consumption. Why?
  2. Consumption, \( c_{it} \), and leisure, \( 1 - n_{it} \), move hand-in-hand. Why?

Combining 1 and 2: Tightening of constraint increases labor supply. Why?
### Preferences

- **Preferences:** \( u(c, n) = \frac{c^{1-\gamma}}{1-\gamma} + \psi \frac{(1-n)^{1-\eta}}{1-\eta} \)

- **Potential issue:** \( \phi \) captures consumer credit, excludes mortgage debt.

### Calibration and numerical simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Explanation</th>
<th>Value</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>Discount factor</td>
<td>0.9713</td>
<td>Interest rate ( r = 2.5% )</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Coefficient of relative risk aversion</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>( \eta )</td>
<td>Curvature of utility from leisure</td>
<td>1.88</td>
<td>Average Frisch elasticity = 1</td>
</tr>
<tr>
<td>( \psi )</td>
<td>Coefficient on leisure in utility</td>
<td>12.48</td>
<td>Average hours worked = 0.4</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Persistence of productivity shock</td>
<td>0.967</td>
<td>Persistence of wage process in Floden and Lindé (2001)</td>
</tr>
<tr>
<td>( \sigma_e )</td>
<td>Variance of productivity shock</td>
<td>0.017</td>
<td>Variance of wage process in Floden and Lindé (2001)</td>
</tr>
<tr>
<td>( \pi_{e,u} )</td>
<td>Transition to unemployment</td>
<td>0.057</td>
<td>Shimer (2005)</td>
</tr>
<tr>
<td>( \pi_{u,e} )</td>
<td>Transition to employment</td>
<td>0.882</td>
<td>Shimer (2005)</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Unemployment benefit</td>
<td>0.10</td>
<td>40% of average labor income</td>
</tr>
<tr>
<td>( \phi )</td>
<td>Borrowing limit</td>
<td>1.04</td>
<td>Debt-to-GDP ratio of 0.18</td>
</tr>
<tr>
<td>( B )</td>
<td>Bond supply</td>
<td>1.60</td>
<td>Liquid-assets-to-GDP ratio of 1.78</td>
</tr>
</tbody>
</table>

*Note: The quantities \( \nu, \phi \) and \( B \) are expressed in terms of yearly aggregate output. See the text for details on the targets.*
Steady-state policy functions

- Units are in terms of steady state yearly aggregate output.
- Higher wage generates income and substitution effect for labor supply.
Consider a permanent decrease in borrowing limit, $\phi = 1.04$ to $\phi' = 0.58$.

- Corresponds to 10 pp drop in debt/GDP ratio (18% to 8%).
- Modeled with linear adjustment path $\phi_t = \max(\phi', \phi - \Delta t)$.

- With adjustment in 6 quarters, no household forced into default (tractability).

Equilibrium solved numerically for the transition between two steady-states. How?
Output is in percent deviation from initial steady state.

Main results: Overshooting of the interest rate and output.
Understanding the interest rate response

- The solid line is the old ss, the dashed line is the new ss.
- The old distribution is a mean preserving spread of the new. Why?
- What would happen if economy immediately jumped to the new ss?
Evidence for overshooting?

5-Year Treasury Inflation-Indexed Security, Constant Maturity (DFII5)
Source: Board of Governors of the Federal Reserve System

Shaded areas indicate US recessions.
2012 research.stlouisfed.org
Output dynamics best understood by breaking into two groups:

- **Individuals near the constraint:** *Income tracking.*
  - At new ss: *Consumption drops considerably.* Labor supply increases. But small effect for aggregate labor since not very productive.
  - Transition: Consumption (and labor) largely determined by $\phi$, not responsive to $r$.

- **Individuals far from the constraint:** *PIH.*
  - At new ss: Consumption increases slightly (lower $r^{ss}$ dominates the precautionary motive), labor supply decreases.
  - Transition: Drop in $r$ increases consumption and decreases labor. **Labor responds more strongly** (since low intertemporal elasticity, $1/\gamma = 1/4$, and relatively high Frish elasticity, 1).

Thus, output initially overshoots and then converges to a lower ss level.
The channel for the output response seems unrealistic

Key insight: Labor supply more responsive to $r$ than their consumption.

- In ss: Credit crunch translates into lower output.
- In transition: Overshooting of $r$ translates into overshooting of output.

The conclusion (that credit crunch lowers output/employment) sounds right, but the channel is not compelling:

- Mechanism relies on voluntary unemployment by the unconstrained.
  - 11% voluntary unemployment?
  - During the crisis, poor (i.e., the constrained) lost more jobs than rich.
- Relies on labor supply strongly responding to $r$. Micro-evidence?
- Generates too small a recession (1% drop in output).
AD channel will offer a more realistic explanation

GL also note:
“Our model is able to generate a recession even with perfectly competitive goods and labor markets. Clearly, adding frictions on the supply side of the model can help in getting a more realistic picture of the effects of a credit crunch on aggregate activity and possibly on unemployment. The introduction of nominal rigidities in the next section is a step in that direction.”

We will come back to the AD channel and nominal rigidities later in the course.
Consumption in general equilibrium with borrowing constraints.

  - Endogenous buffer-stock behavior (restricted asset supply).
  - Interest rate lower than complete markets benchmark. Further lowered by credit crunch.

- Guerrieri and Lorenzoni (1994): Transition to tight credit.
  - Overshooting for $r$. Quantitative importance for the recent liquidity trap.
  - Some output/employment response through neoclassical channels. But points to the AD channel for a better understanding.

Tightening of credit exogenous. Presumably linked to house prices.

- **Next time**: Borrowing constraints and asset prices.